Application of Wavelet Theory in Image Processing: A Study of Digital Image Denoising

A thesis submitted in partial fulfilment of requirements for the degree of

DOCTOR OF PHILOSOPHY

in

MATHEMATICS

by

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Hyderabad, India-500032

(2022)

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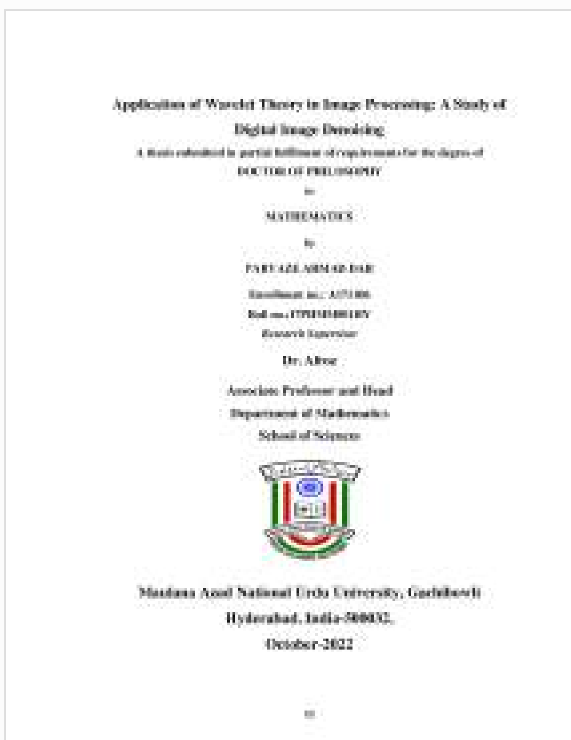

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Parvaze Ahmad Dar

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List of Symbols

S no.	Symbol	Name
1	ψ	Psi
2	ϕ	Phi
3	α	Alpha
4	β	Beta
5	ε	Epsilon
6	λ	Lambda
7	θ	Theta
8	$\sqrt{\ }$	Square Root
9	$\sum$	Summation
10	$\int$	Integral
11	η	Eta
12	δ	Delta
13	ω	Omega
14	τ	Tau
15	π	Phi
18	$\lvert x\rvert$	Absolute value of x
19	$\lVert x\rVert$	Norm of x

Abbreviations

S. no.	Abbreviations	Words
01	AIP	Analog Image Processing
02	AWGN	Additive White Gaussian Noise
03	CWT	Continuous Wavelet Transform
04	DIP	Digital Image Processing
05	DWT	Discrete Wavelet Transform
06	FT	Fourier Transform
07	GSI	Gray Scale Image
08	HPF	High Pass Filter
09	LPF	Low Pass Filter
10	IWT	Inverse Wavelet Transform
11	ICWT	Inverse Continuous Wavelet Transform
12	IDWT	Inverse Discrete Wavelet Transform
13	IWPT	Inverse Wavelet Packet Transform
14	MSE	Mean Square Error
15	MATLAB	Matrix Laboratory
16	PSNR	Peak Signal to Noise Ratio
17	SNR	Signal to Noise Ratio
18	SPN	Salt & Pepper Noise
19	STFT	Short Time Fourier Transform
20	WT	Wavelet Transform

Chapter 1

INTRODUCTION

1.1 Introduction

In the mathematical world, the transformation of one function to another for analysis purposes is an old fashion. Every signal cannot be analyzed in their real domain. Therefore, for analysis and observation purposes or to know the nature of a signal, one should have to change it to another form or transform it into another domain. In other words, we can say that the transformation of any signal into another form helps us to investigate or to manipulate it. These mathematical transformations make it easy way for a learner to know the nature and purpose of these signals and how to deal with them as well.

There are several types of signals available for analysis purposes, such as seismic signal, audio signals, speech signals, astronomical signals, ECG signals, medical imaging. In practice, it is known that the maximum numbers of signals have a raw shape. In some signals, the signal frequency is essential and a vast amount of information is hidden in that part. To obtain such information, the Fourier Transform is applied to mathematically transform it from time-based to frequency-based by decomposing it into sinusoids of varied frequencies. There is no doubt that the Fourier Transform (FT) has its own beauty and importance, but it has a big drawback: it loses time information while transformation to the frequency domain. In other words, we cannot say at what time which part of the signal occurs. A Short-Time Fourier Transform (STFT) came in existence, which carried out an approach called the windowing of a signal. Therefore, with the help of STFT, only a small part of the signal can be analyzed. Of course, STFT provides information about the time at which the frequencies of the signal occur.

It also has a serious drawback: once a particular size is fixed, the window size cannot be changed and remains fixed for all frequencies. Moreover, while performing STFT on the signal, if we fix a small size for a window, it gives better time-resolution, but poor frequency-resolution. On another side, if we fix a large size for a window, it gives poor time-frequency-resolution and better frequency-resolution. In this way, different techniques and algorithms, like Discrete Fourier Transform, Fast Fourier Transform, Wavelet Transformation, and Curvelet Transformation are utilized for signals analysis.

Usually, signals carry other unwanted signals, such as noise, and it is a challenging task for modern science to remove them without affecting their original properties. Sometimes, we say noise-contaminated in the signal are components of the same signal unidentified and unrecognized, and do not match its single component to the properties of the actual signal. The noise available in a signal may affect the whole or part of the signal. The removal of noise from noisy signals for signal enhancement and quality improvement depends on the technique and method applied.

Signals such as seismic signals, astronomical signals, speech signals, audio signals, ECG signals are major sources of information about different branches of science. Here, we discuss an essential signal known as Digital Image. Digital images are considered a rich source of information, available in almost every field of human life. Digital image processing is a process of image enhancement, like image denoising, image compression, and diagnosis of medical images through computers. Digital images catch noise during the process of capturing, transmitting, processing, and many other reasons. This noise may harm some components or features of the entire or part of the image. Therefore, for the removal of such types of noise, different approaches, algorithms, techniques, and methods have been adopted to visualize the original purpose of such types of digital images.

1.2 Literature Review

A review of literature is nothing, but a comprehensive study or data related to any topic or research. This data may be available in different forms like books, newspapers, magazine cuttings, research papers, thesis, dissertations, audio & video, etc. In the same manner, by taking all things into the consideration, we have mentioned the literature review for this research.

Sami Hussien Ismael, Firas Mahmood Mustaf, and İbrahim Taner in their research paper titled, "A New Approach of Image Denoising Based on Discrete Wavelet Transform"**,** propose a method for digital image denoising based on wavelet transform (WT), because it has a quality of splitting the digital image into sub-bands and then work on each sub-band frequencies individually. They also used a median estimator to estimate the noise ratio of noisy images. In the end, it shows that the proposed approach gives good values for image denoising in terms of Mean Square Error (MSE) and Peak-Signal to Noise-Ratio (PSNR)[1].

Barware Mela Ferzo, Firas Mahmood Mustafa Nawroz, in their research article entitled, "Image Denoising in Wavelet Domain Based on Thresholding with Applying Wiener Filter" , proposed a new method for image denoising for the images affected by additive white gaussian noise (AWGN), by applying Wiener filter before and after the wavelet transform. For the removal of noise from the pixel, two-dimensional discrete wavelet transform, as well as threshold techniques and wiener filter, is taken into consideration. Finally, a two-dimensional inverse discrete wavelet transform (2D-IDWT) is used to remove noise and get the objective. The performance of the method in terms of image denoising is shown by the peak signal-to-noise ratio (PSNR) and gives an improvement of about **17.5%** in comparison with the results of the related work[2].

Ura Tuba, Dejan Zivkovic, in their article with the title, "Image Denoising by Discrete Wavelet Transform with Edge Preservation", proposed a method for denoising digital images. They combine three methods that were applied in the wavelet domain to improve the performance of image denoising. Therefore, for the image noise removal, the soft thresholding approach was combined with the median filter. The method was tested on four benchmark images. Finally compares the results of the proposed method with other literature methods in terms of peak signal-to-noise ratio (PSNR). The proposed method also achieves robust results and is efficient for removing Gaussian noise[3].

Nalin Kumar and Mrs. M Nachamai, provide noise removal method in their research article entitled, "**Noise Removal and Filtering Techniques used in Medical Images".** In this paper, they primarily used three filters namely Weiner, Gaussian, and median filters which were already used successfully in medical like Magnetic Resonance Image **(MRI)**. The digital image MRI is usually affected by the noises like Salt and Pepper, Speckle, Gaussian, and Poisson noise. The method was applied to each type of image, which was contaminated with the noises mentioned above. The medical image taken into consideration for operation includes grayscale, RGB, and MRI images. The test of these algorithms is done on the bases of measuring the size of the file, histogram, and clarity of the image. Finally according to the experimental results median filter performs better in terms of removing salt-and-pepper noise and Poisson noise for images in grayscale, the Weiner filter performs better for removing speckle and gaussian noise, and the Gaussian filter for the blurred noise[4].

QingKum Song, Li Ma, JianKun Cao, and Xiao Han have proposed an approach in their research article with the title, "Image Denoising Based on Mean Filter and Wavelet Transform", the authors combine two filtering methods for digital image noise removal processing. The result and experiment test effect for the noise reduction of noisy images seem superior to the other simple denoising methods. The result and efficiency of the method for denoising of images were checked by Signal-to-Noise Ratio (SNR) and Root Square Error (RSE)[5].

JunLie Song, MeiJuan Chen, Chang Jiang, presents a new threshold function in their research article entitled, "Research on Image Denoising Method Based on Wavelet Transform", which is based on traditional soft & hard threshold functions for image denoising. Through the software of MATLAB, the denoising effect of soft threshold, hard threshold, and the proposed threshold function are compared in terms of signal-to-noise ratio (SNR) and root mean square error (RMSE). Finally, on the result of the base,it shows that the result of the proposed threshold function has a higher signal-to-noise ratio and small mean square error i.e. SNR=26.27709 and MSE=153.4579 [6].

Y. Yang, Z. Su, L. Sun, in their research paper titled**,** "Medical image enhancement algorithm based on wavelet transform”**,** proposed a better method for medical image denoising. The medical image is decomposed by wavelet transform, high-frequency sub-images are decomposed by haar wavelet, noise in the frequencies was removed by the soft-threshold method, and high-frequency coefficients were enhanced by diverse weight values in different sub-images. Finally, enhancement of images was carried by inverse wavelet transform especially haar wavelet transform. The result of the experiment visualizes that the method is not only better for image enhancement but can preserve image edge features[7].

Wasan A. Alawsi, Zahraa Ch. Oleiwi, Ali H. Alwan, and Maytham K. Fadhil, in their article entitled, “Performance Analysis of Noise Removal Techniques for Digital Images”**,** gives an overview of various sources of noises, and noise removal techniques for digital images. The main motive of this paper was to perform segmentation of the corrupted image before using a noise removal filter and the noise filters are applied to eliminate noise for each segment of the image individually. A comparative study between noise removal filtering and segmentation-based method for image denoising is produced in terms of MSE and PSNR and then compared the result with the original image. For the same operation, six image samples are used for the test for the image noise removal approach. The segmentation technique has shown the best result in terms of MSE and PSNR in comparison with traditional noise removal filters[8].

Tanay Mehendale, Vishal Ramina, and Soham Pinge, in their research article entitled, “Analysis of the Effects of Different Types of Noises and Wavelets used in Denoising of an Image using Wavelet Transform”, uses the latest method for digital image denoising through wavelet transformation to eliminate much more noise and obtain minute- noisy image by suppressing noise. Two main factors may affect the image, first result-type of wavelets and the second is the type of noise. In the same paper, these factors are studied and detailed analyses are given so that to select the correct parameter while doing image denoising by wavelet transform[9].

Leena Jain, Palwinder Singh, in their research article entitled, “A novel wavelet thresholding rule for speckle reduction from ultrasound images”, proposed an important thresholding technique based on wavelet transform. The main aim is to reduce speckle noise from ultrasound images because speckle-noise causes blur or loss of edges and may degrade whole or important features of the image. The wavelet transform performs multi-scale analyses of the signal by treating different frequency components present in an image separately. Finally, the testing result of the proposed thresholding technique provides superior results for speckle noise reduction, edge preservation, and feature preservation of medical ultrasound in comparison to existing thresholding techniques[10].

Chintada Lavanya, D. Yugandhar in their article entitled**,** “Medical Image Enhancement Based on Wavelet Transform, proposed” a blind image de-convolution (BID) method to protect mammographic images from gaussian bluer, and real dual-tree wavelet transform is used for image enhancement. The output results obtained from the experiment are compared with the hybrid combination of BID, conventional wavelet Transform, and median filter approach. So that to eliminate gaussian noise, Salt & Pepper and Speckle Noise. The performance evaluation of the method is obtained by the platform of the Mini-MIAS database[11].

Nasser Edinne Benhassine, Abdelnour Boukaache, and Djalil Boudjehem, in their research article entitled, "Medical image denoising using optimal thresholding of wavelet coefficients with selection of the best decomposition level and mother wavelet"**,** proposed an approach for medical image denoising based on discrete wavelet transform by selecting best decomposition level of the mother wavelet**.** Then thresholding technique is applied on detail coefficients**.** Optimal thresholding is obtained by using optimization techniques such as the crow search algorithm and social spider optimization technique. In the end inverse wavelet transform (IDWT) is applied to reconstruct the denoised image and the results of the approach are visualized on the bases of peak signal-to-noise ratio, mean square error, and the structural similarity index measure. The investigational results visualize that the proposed denoising method is better than the standard methods[12].

Bachir Dehda, Khaled Melkemi, in their research article entitled, "Image Denoising using new Wavelet Thresholding Function"**,** proposed a new digital image denoising approach based on the thresholding technique. In the same approach, the authors introduce a nonlinear thresholding function distinguished by a shape parameter and properties. Such a type of uniqueness makes the proposed method competent to attain a compromise between both traditional thresholding techniques such as Hard and Soft thresholding. The experimental results of many images on test bases are framed in terms of the Peak Signal to Noise Ratio (PSNR). Finally, the computed results visualize that the proposed method provides better performance as compared to other classical thresholding approaches in terms of the appearance quality of the noise-free image and has higher PSNR[13].

Yang Zhang, Weifeng Ding, Zhifang Pan, and jing Qing, in their research article entitled, "Improved Wavelet Threshold for Image De-noising, proposed an approach for digital image noise removal to get higher-quality images". An image gets noise during collection, transmission, and storage, decreasing the real quality of the image. Therefore noise reduction process is important and necessary to obtain a noise-free image. A wavelet transform is a powerful tool in the field of image denoising, because of its multi-analysis property, relativity removal, low entropy, and flexible bases. Shortcomings in thresholding like hard threshold function are discontinuous, soft threshold function causes constant deviation and traditional threshold functions have some lacking in image de-noising. Taking all things into consideration, the authors proposed a method for noise removal in digital images. In the first step, the method decomposes the noisy digital image to determine wavelet coefficients. In the next step, these coefficients are used on the high-frequency part for threshold processing utilizing a superior threshold function. In the end, de-noised images were acquired to reconstruct the images under the estimation in the wavelet-based conditions. Acquired outputs illustrate that this method is better than traditional hard threshold de-noising and soft threshold de-noising methods, in terms of denoising performance and visual effects[14].

Diwakar Manoj, Chauhan Diwaka, Tejeswi Negi, in their article entitled, "An Enhanced MoBayesShrink Thresholding for Medical Image Denoising", proposed a method that controls the threshold (T) adaptively to remove noise. The proposed method is applied to numerous grayscale test images with sizes (512 × 512) by using Daubechies (db4) at four levels of decomposition. Bayes method is modified so that noise over the edges can be reduced exactly and edges can be better preserved. Method result Performance is obtained using the tool peak signal to noise ratio (PSNR). The output results of the proposed method are better than the existing denoising algorithm in terms of noise removal and edge preservation, such as NSTISM (19), SPBIDM (18), BayesShrink, NormalShrink, and Modified BayesShrink[15].

Arun Dixit, Poonam Sharma, in their research article entitled, “A Comparative Study of Wavelet Thresholding for Image Denoising”**,** has reviewed different methods of image denoising using wavelet thresholding. A brief analysis of different wavelet-based image denoising methods like VisuShrink, SureShrink, BayesShrink, ProbShrink, BlockShrink, and Neigh ShrinkSure is done. Then these wavelet-based approaches are compared with other special applied methods i.e. Median Filter and wiener Filter. Finally, on the examination bases, wavelet-based methods like Prob Shrink, Block Shrink, and Neigh Shrink Sure perform superior to spatial domain methods. The result of each method is examined and compared on the base of Peak Signal to Noise Ratio (PSNR) as well as on image appearance superiority bases[16].

Ridhi Bhatnagar, Aparna Vyas, in their article entitled, “Comparative Study of Wavelet Transform and Wavelet Packet Transformation in Image Denoising Using Thresholding Technique”**,** are dealing with the comparative study of different methods for digital image denoising by using wavelet transformation and wavelet packet transformation (Coiflet, Haar, Daubechies, Symlet wavelet). In the modern world images are considered as the best way of communication for the people of any field, but the noises present in images may affect little or whole information present in that images.

The images used in the experiment are contaminated with Gaussian noise, then applying thresholding approach to provide the best result and gain back the original information of the image. The comparative results are shown or revealed by Peak signal-to-noise ratio (PSNR) and mean square value (MSE) formulas[17].

P. Venkata Lavanya, C. Venkata Narasimhulu and K. Satya Prasad, in their research article entitled, “Denoising Images by Dual-Tree Complex Wavelet Transform Combined With Meta-Heuristic Optimization Algorithms”, present a Duel-Tree Complex Wavelet transform (DTCWT) approach to denoise image because it performs Multi-Resolution decomposition by two DWT trees. The wavelet techniques, Soft, and hard thresholding are used to threshold wavelet coefficients. The proposed research provides a new technique for the removal of noise from noisy images and also provides clear information about an image through thresholding and optimization techniques. The process of optimization is taken by various Meta-heuristic Optimization Algorithms, Genetic Algorithms (GA), and Grey-wolf optimization (GWO) algorithms. The threshold value is applied after the Bayesian method. The result obtained from the proposed method was better when it is compared to the results of other techniques like VisuShrink, SureShrink, and BayesShrink. The performance of methods in terms of image denoising is decided on the bases of Peak Signal to Noise Ratio (PSNR), Mean Square Error (MSE), and present visibility or quality of the image[18].

Payal Gupta and Amit Garg, in their research paper entitled**,** “Image Denoising Using Bayes Shrink Method Based on Wavelet Transform”**,** have proposed the best method for removal of noise from Ultrasound images, based on linear filtering, for the images which were corrupted by gaussian noise. The proposed method consists of linear filtering of appropriate wavelet coefficients of the image, corresponding to diagonal and vertical details. For experiment and test purposes standard Boat images and real US images are taken into consideration. The proposed method shows good performance and effectiveness for the noise removal of the above images. Finally, results are framed and compared in terms of PSNR and SSIM values obtained for gaussian corrupted images. In the end, the results of the proposed method showed that the method is more efficient and direct to the enhancement of parameters PSNR and SSIM[19].

Khawla Bnou, Said Raghay, and Abdelilah Hakim, in their article entitled, A Wavelet Denoising Approach Based on Unsupervised Learning Model, proposes the best wavelet denoising approach based on the unsupervised learning model. Images are known as well sources of information, but the unwanted thing which makes them polluted is called noise. Therefore, image denoising plays an important role in image processing, because it helps to clean images from noisy images. The proposed method is based on wavelet transformation, because of its Sparsity, multi-resolution structure, similarity with the human visual system, and has the property of cleaning or denoising images in a good manner. By using the k-singular value decomposition (K-SVD) algorithm, they obtain an adaptive dictionary by learning over the wavelet decomposition of the noisy image. Finally, on the bases of analyses and results on benchmark test images, it clearly shows that the proposed method achieves the best results in terms of digital image denoising. The denoising performance of the said proposed method is better than some other well-developed denoising methods; it is visualized in terms of peak signal-to-noise ratio [20].

1.3 Image and Its Types

1.3.1 Image

The name of the image came into existence from the Latin word 'Imago'**,** which means a copy of anything. An image is an actual appearance or photo or two-dimensional picture of any place, person, or thing. In signal processing, it is a digital representation of physical quantity. In the modern world, images are considered the biggest source of information. A huge amount of data in the form of images are captured and transformed daily for different information purposes.

Mathematically, the image is a two-dimensional function $f(x, y)\ \ x, y \in Z$. Here (x,y) are special coordinates. The amplitude of $f(x, y)$ at any pair of coordinates (x,y) is called the intensity at that point or pixel value. The 2-D function $f(x, y)$ is called a digital image if its values i.e. x, y, and amplitude are finite[21].

1.3.1.1 Pixel

A pixel is an important and small component of an image, having a particular place in the image. Usually, we called it the intensity value of the image[21].

1.3.2 Digital Image

The image which is captured by digital device (camera) is called the digital image. Examples are images captured by the digital camera, computers, CCTV-type images, etc. In other word image obtained by a digital device which can be processed by different computers are known as digital images. These images are stored in binary codes and are framed mathematically as a matrix of pixels. Examples are satellite camera images, modern medical camera images, scientific camera images, human purpose camera images, etc[22].

1.3.3 Colour Image

A picture or photo displayed with different colors with the help of a computer device is known as a color image. In other words, an image that contains three color channels, defined in different ways is called a color image. The examples are RGB (red, green, blue), HIS (Hue, saturation, intensity) etc[22],[23],[24]. Examples of color images;

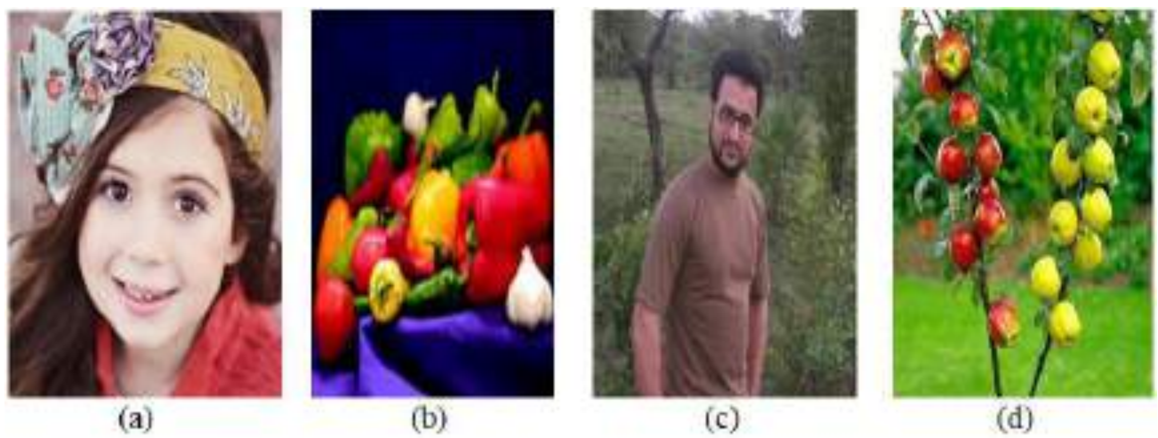

Fig. 1.1: Colour (RGB) Images; (a). Pouts (b). Vegetables, (c). Author and (d). Apples,

1.3.4 Grayscale Image

An image whose each pixel value represents the intensity information of light i.e. it displays only bright-white and dark-black colors. In other words, we can say that the images contain darkest-black, brightest-white, and gray colors, but the gray color has multiple color levels. Usually, grayscale images have 256 gray shades or pixel values ranging from 0 to 255. Here the value zero, or the pixel value nearest to zero, denotes dark pixels or dark shade and the pixel values nearest to 255 denote the white pixel or white shade[21],[22].Examples of grayscale images;

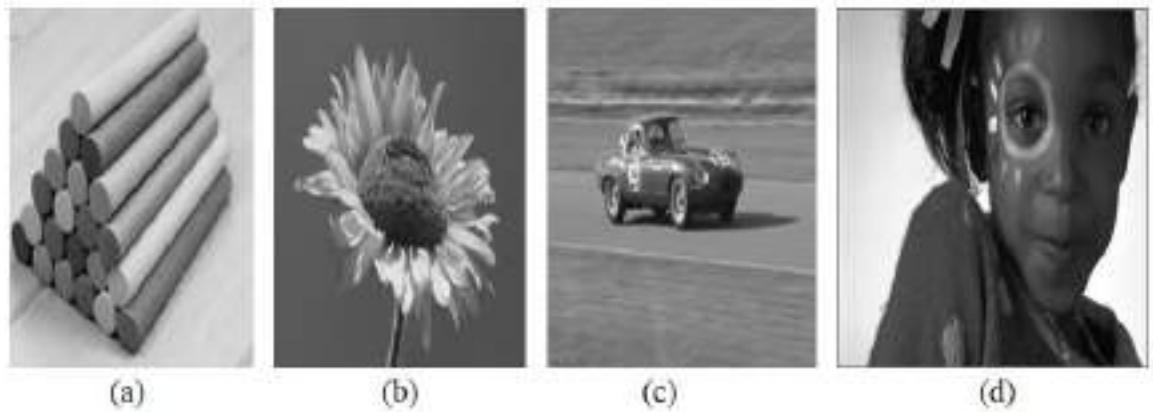

Fig.1.2: Grayscale Images (a).Chalks, (b). Sunflower, (c). Motor Car and (d). Girl

1.3.5 Binary Image

If an image has only two intensity (pixels) values i.e. 0 and 1, we call it a binary image. Here 0 is displayed as a black pixel and either 1 is displayed as a white pixel[22]. Example of digital binary image;

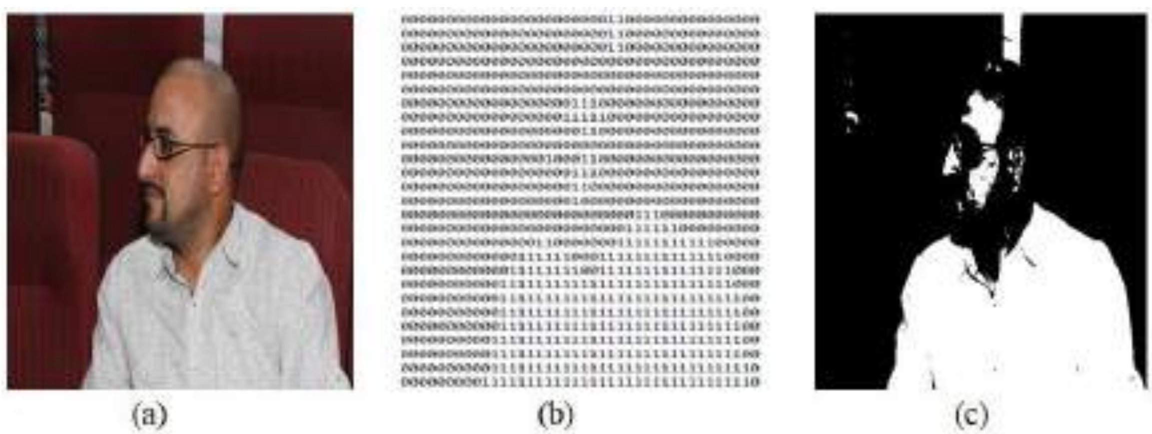

Fig.1.3*:* (a). Author RGB Image, (b). Image Matrix and (c). Binary Digital Image

1.4 Image Format

An image can be stored in many available formats. Some commonly used and available formats for image storage are;

1.4.1 Joint Photographic Expert Group(JPEG)

JPEG images are digital images, which are compressed to store a vast amount of data

(Information) in a little size file. That is why most of the common people used this type of format because they store massive data in the form of photos in this format[24].

1.4.2 Tagged Image File Format (TIFF)

TIFF images are uncompressed digital images. TIFF contains detailed image information and takes large file sizes. These images are very flexible in the matter of color content. This type of format is usually used in photo software and photo layout software like, “Photo-Shop and Quark Design”[24].

1.4.3 Graphic Interchange Format (GIF)

JIF format of digital images can compress the images but is not more effective than JPEG format. This type of compression of image is lossless i.e. no information or details about the image is lost. Therefore, the file size of GIF cannot be lesser than the JPEG file size[24]

1.4.4 Portable Network Graphics (PNG)

PNG was introduced to replace the GIF format because the copyright of GIF was in the possession of another company and no one wants to pay the licensing fee for the same format. This format is able for excellent compression and full-color range.

1.4.5 Raw Image Format (RIF)

RIF contained a source of data from a digital camera. It is called raw image because the images have not been processed and are still unedited and unprinted. The size of the RIF is typically large because the raw photos or files include a huge amount of data uncompressed. That is why they are converted into TIFF or another format before editing and color processing.

1.5 Description of Digital Image

In the above, we have already mentioned that the image is a 2-D signal or matrix $f(x, y)$, with coordinates x and y together forming pixel values of the same image. The digital image is nothing but a structure of different intensity values.

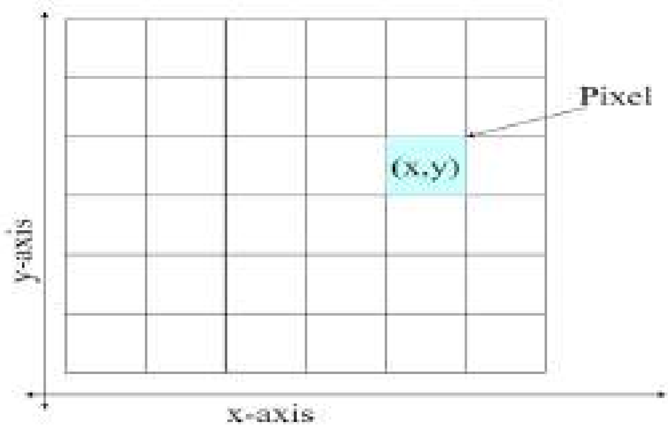

Fig. 1.4: 2-D Pixel Matrix Representation of Digital Image

1.6 Graphical Representation of The Digital Image

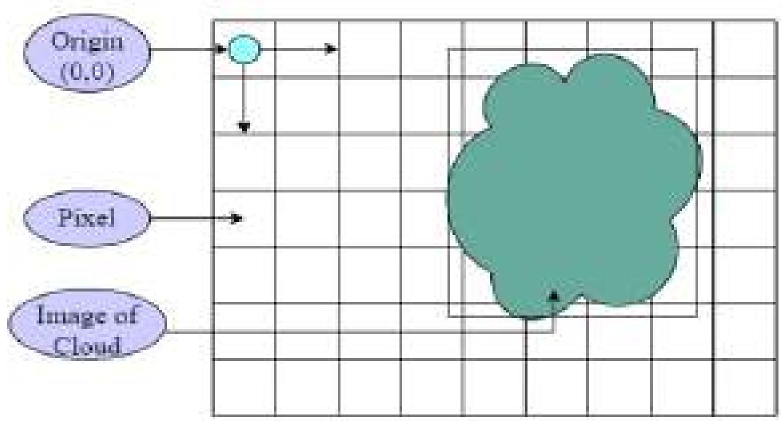

Fig.1.5: Bitmap Representation of Digital Image

1.7 Mathematical Representation of The Digital Image

A digital image is a 2-D function mathematically defined as;

$$f(x,y) = \begin{bmatrix} f(0,0), f(0,1), & --- & --- & f(0,n-1) \\ f(1,0), f(1,1), & --- & --- & f(1,n-1) \\ --- \quad --- & --- & --- & --- \\ --- \quad --- & --- & --- & --- \\ f(m-1,0), f(m-1,1), & & --- & f(m-1,n-1) \end{bmatrix}_{M \times N} \quad .... (1.1)$$

Here M and N are the corresponding rows and columns of the digital image

1.8 Image Processing and Its Types

Image processing is a process or a way to carry out an operation on images to achieve an improved image. Sometimes we call it a method applied for analyzing and manipulating the images. Image processing is a sub-type of signal processing. In image processing, the input data is an image, but the output image may be an image or some characteristics or features related to the input image. In modern technology and research, image processing has spread its wings in several areas like engineering, mathematics, and computer science disciplines. Image processing can be done in two ways[22],[24].

1.8.1 Digital Image Processing

Digital image processing is a process or method, which assists in enhancing and improving digital images with the help of computers. There are three steps under DIP works for image enhancement[22],[24];

a) Pre-Processing

b) Enhancement

c) Present Information Removal

1.8.2 Analog Image Processing

Analog image processing is a technique or method, usually applicable for hard copies like Printouts and Photographs. In other words, AIP is useful for analog 2-D signals. In AIP electrical signals are used for image manipulation. The analog signal can be both periodic and non-periodic[24].

1.9 Component of the Digital Image Processing

The first and most important component of digital image processing is the Camera. It is used to capture an image of a 3-D object and depict or represent it on 2-D paper.

1.9.1 Digital Camera

The digital camera is a camera, which produces a digital image that can be displayed, shared, and stored on the computer. There are two types of sensors used in the digital camera. The first one is called Charged Coupled Device (CCD) and the second one is called Complementary Metal-Oxide Semiconductor (CMOS). The CCD sensor-type cameras comprise a huge number of tiny photo-diodes, known as Photosites. The CMOS sensor type camera has a big number of transistors are used for amplification of the image signal at each pixel location. Sometimes these two sensors were responsible for noise creation in the digital images.

The digital camera can capture a digital image at dissimilar resolutions like 230×388, 410×330, and 515×750,710×615, etc pixels on the little-to-average resolution range to 1250×916 or 1500×1300 on the higher resolution size. An ordinary digital camera can produce around 16 million colors. In other words, for each intensity (or pixel) value, there are 16 million colors available.

1.10 Objective of Digital Image Processing

One of the main objective of DIP is to convert the input image into a digital shape and then operate some operations on it and then take out various valuable information from it [22],[24].

1.11 Sources of Digital Images

The electromagnetic energy spectrum is one of the main sources of digital images;

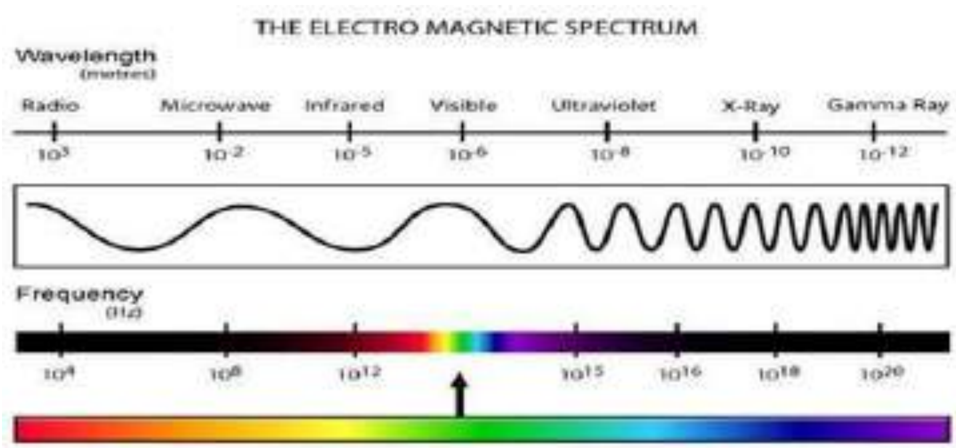

Fig.1.6: Electromagnetic Energy Spectrum for Image (online)

1.12 Mathematical Preliminary

1.12.1 Convolution

Mathematically, the convolution of two functions $g_1(t)$ and $g_2(t)$ is given as[25];

$$g(t) = \int_{-\infty}^{-\infty} g_1(x) g_2(t-x) dx \qquad \text{............. (1.2)}$$

Symbolically we write,

$$g(t) = g_1(t) * g_2(t) \qquad \text{.............. (1.3)}$$

Here symbol * denotes the convolution of two functions

1.12.2 Parseval's Identity

If two functions**,** $g_1(t)$ and $g_2(t)$ are related to their Fourier Transforms

$g_1(\omega)$**,** and $g_2(\omega)$

Then their identity is given as;

$$\langle g_1(t), g_2(t)\rangle = \frac{1}{2\pi}\langle g_1(t), g_2(t)\rangle \qquad \text{..............} \ (1.4)$$

If $g_1(t) = g_2(t)$, it is called Parseval's theorem.

1.12.3 Mathematical Signal Transformation with Application

As usual, signals carry a lot of information. Sometimes information of signals may be defective or in raw data form. In the beginning, signals were investigated and analyzed through time domain-based methods. These techniques and methods were not enough to study signals, because all characteristics of the signal were not studied. Also, we know that the signal frequency contents hide huge information, Therefore, there was a of need some proper investigating techniques and methods, in order to get original and real information from it. Then a robust method came into existence in the shape of the Fourier Transform to analyze these signals. This approach for signal analyses came into practice in 1807 by the French geographer and mathematician Joseph Fourier. Through this method, the original signal is transformed from time-based to frequency-based, so that information can be taken from frequency contents. In other words, the FT decomposes the given raw signal into sinusoids of different frequency components. Then information about the given signals is taken from these frequency components. The mathematical form of the Fourier Transform (FT) is given as;

$$F\left[f(t)\right]=\int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt = F(\omega) \quad \text{, here } \omega = 2\pi\nu \ or \frac{2\pi}{T}$$ .... (1.5)

In the above function, F denotes FT, $f(t)$ is an analyzing signal. T is called a period which represents cycle length in a particular time, ν is called ordinary frequency, t is a time, and ω is the angular frequency. The integral (continuous sum) used in Fourier transform utilizes the properties of sine and cosine series to improve the amplitude from minus infinity to plus infinity. The Inverse Fourier Transform (I.FT) is given as;

$$f(t)=\int_{-\infty}^{+\infty} F\left[f(t)\right]e^{i\omega t}d\omega$$ .... (1.6)

Here a major problem is connected with FT, i.e. the time information is not available when transforming to frequency information. In other words, we can say that, while doing FT of any signal, we can't say in what time which frequency content occurs. In any signal, there are amplitudes of different frequency components present. Therefore, the frequency resolution of the time domain is very high and has zero time resolution. Especially, FT is appropriate for stationary signals and is not suitable for non-stationary signals, because frequency changes occur over time. Due to some reasons, FT is not appropriate for analyzing some signals and has some disadvantages and limitations like;

- It is not applicable for stationary signals
- The Fourier Transform is unable to confirm the local behaviour of the signal i.e. it cannot check any discontinuity or spike in an analyzing signal.
- It has a zero-time resolution, but extremely high-frequency resolution.

- In Fourier Transform the analyzing (input) signal may be real or complex, but the reconstructed (output) signal is always a complex function.

Because of these disadvantages and limitations, there was a requirement for a new method that will solve the problem created in FT.

Then an approach or technique came into existence in the shape of Short Time Fourier Transform (STFT) to resolve the problems of Fourier transform. The concept of STFT was introduced by Dennis Gabor in 1947. In this approach, we simply multiply analyzing signals with the window function. The concept behind the STFT was to set a window to break down the signal into different parts and obtain time localization. The window here used in the transformation has a small length, but a fixed window size. Here partitioning of signal simply means, windowing the signal by multiplying the window function $\omega(t)$ by the original function $f(t)$. Then FT is applied to each part of the signal which was decomposed (sliced) by STFT. The mathematical representation of STFT is given below;

$$F(\kappa,\upsilon)=\int_{-\infty}^{-\infty} f(\kappa,t)e^{-i\upsilon t}dt=\int_{-\infty}^{+\infty} f(t)W(t-\kappa)e^{-i\upsilon t}dt=\left\langle f(t),g_{\kappa,\upsilon}(t)\right\rangle \quad \text{.............. (1.7)}$$

and $g_{\kappa,\upsilon}(t)=W(t-\kappa)e^{-i\upsilon t}dt$ .... (1.8)

Here, υ is called the average frequency of the window function, κ is the centre of the window in time $f(\kappa,t)$ is part of the signal $f(t)$ and $g_{\kappa,\upsilon}(t)$ is known as the short-time Fourier Transform Atom.

One of the key concepts behind the Short Time Fourier Transform is that it considers the non-stationary signal as stationary for a short time interval. Therefore, STFT gives the solutions to the problems of FT under some limits. For any signal, STFT provides three

3-D information i.e. about time, frequency, and amplitude. One of the big weaknesses of the STFT is that the size of the window remains always fixed. But the constant size of the window is not acceptable for all frequency components present in the signal.

To resolve the problem related to STFT, another transformation came into existence in the early 1980s known as Wavelet Transform (WT). Wavelet theory is well suited for digital signals in terms of analyzing the local behaviour ("local part of a big signal") of the input signal. Sometimes we say it presents localization in time (space) to a certain degree. In simple words, wavelet series indicates a "Square Integrable Function" with regards to the "Orthogonal set of Bases Function" called wavelets, which means small wave. In the beginning, wavelet theory was used for analyses of seismic signals for time dimension recognition. With time it rapidly spread application in several areas like finance and statistics, turbulence, medical science, astrophysics, image processing, and other signal processing areas. Wavelet Transform decomposes the signal into a set of functions called wavelets (Daughter Wavelets). Then analyze the characteristics of the signal in the modular domain as well as in the time domain (converted-space domain). We will discuss wavelet transformation in detail in the next chapter of this thesis.

At the current time, we are working on the application of wavelet theory in digital image processing. In particular denoising (noise-reduction) of digital images through wavelet transformation. Here we will also discuss the purpose and necessity of digital image denoising (noise removal, noise compression). As we know digital image denoising is one of the important tasks of signal processing.

Digital images are usually corrupted by noises (unwanted signals). Images catch noise during the process of acquisition, transmission, and recovering it from storage. With our naked eyes, we can observe some dots on the digital image; the reason may be low lightening during image capturing. This may be the cause of the original images getting corrupted by noise. These unwanted or noisy signals may corrupt both images and videos. One reason for the image denoising is that a noisy image is not delightful to view. Sometimes the important details in the images are confused with the noise and it is difficult to identify the real information in the image.

Mathematically we can define image denoising as;

$$h(x, y) = f(x, y) + g(x, y) \qquad \text{.............} \; (1.9)$$

Here $h(x, y)$ is a noisy image, $f(x, y)$ is an original image, and $g(x, y)$ is a simulated noise (any type). The main purpose is to estimate $f(x, y)$ given $h(x, y)$.

Several methods and techniques were applied for digital image noise reduction or compression.

1.12.4 Simulated Noises and Its Types

1.12.4.1 Digital Noise in Image Processing

Digital noise in digital images is random pixels spread all over the image. It arises during the process of transmission, capturing at the wrong angle of the camera, or getting it from media storage. It often occurs due to capturing images in low light like, in a dark room, at night, etc. As already mentioned above, noise may enter into the images due to sensors (CCD & CMOS) installed in digital cameras.

1.12.4.2 Digital Noise Types

There are several types of simulated noise, some common noises are mentioned below[26];

1.12.4.2.1 Gaussian Noise

Gaussian noise is usually called additive noise. It arises in the images because of little illumination during the process of capturing or sometimes because of high temperature. In the process of modification of pixels, a certain distribution is added to each pixel. It may be caused because of the noise available in electric circuits.

1.12.4.2.2 Salt & Pepper Noise

This type of noise is also called impulse noise. It affects only a small number of pixels and the rest image pixels remain untouched. It may be caused, because of abrupt disturbances like dust or a faulty CCD during the accusation of the image[26].

1.12.4.2.3 Poisson Noise

It is sometimes called Photon or Short Noise. It may follow the Poisson distribution. It often arises in the lighter parts of an image. It may arise, because of the dissimilarity in the number of photons sensed at a given exposure level[26].

1.12.4.2.4 Speckle Noise

It is also called multiplicative noise. This type of noise may corrupt images like Ultrasound, Laser Sonar, etc. For the modification of the pixels of the image certain distribution is multiplied by certain pixels of the image. There are still number of noises that can be encountered in the digital images.

According to the latest research articles and research trend in digital image processing, especially for digital image denoising, it clearly shows that there is a huge scope of research in the field of digital image denoising. It is also a critical and complex research problem in the application of wavelet theory. So in order to deal with this research problem, it needs to study and apply some approaches, methods & techniques, and other statistical parameters. We need to work on different thresholding techniques, applications of the different wavelet functions, exiting filters and filtering techniques, and the application and validation of other mathematical thoughts for an acceptable reason for the outcome of the proposed work in terms of the digital image denoising. Finally, some modifications and adaptations have been done in the existing work in the opinion of statistical parameters. Different techniques and methods have come into existence from time to time for digital image analysis, enhancement, modification, and other purposes. Fourier transform is an important technique for signal analyses, in which a given signal (or image) is decomposed into sine and cosine waves of the number of frequencies. In the same way, wavelet transform also decomposes the signal (Image) into translated and scaled versions of the wavelet (Mother Wavelet). We have already mentioned above the disadvantage and limitations of Fourier transform and STFT. But now these signals are well analyzed by the latest transformation known as wavelet transformation because it analyzes the given signal into two domains i.e. time domain and frequency domain. In particular, the signals carrying quick changes could be well analyzed by the wavelets, then the existing sinusoids. Therefore, wavelet theory is considered the best tool for analyzing the given signal. Two important functions play a vital role in the Wavelet transformation for the signal analyses, called the wavelet function ψ (or Mother Wavelet) and the scaling function ϕ (Father Wavelet). These two functions together produce a set or family of functions or wavelet functions (Daughter Wavelets). Later this set of functions is implemented to decompose or reconstruct the given signal.

Here we can show graphically the wavelet transform in comparison with other transformations for a given signal.

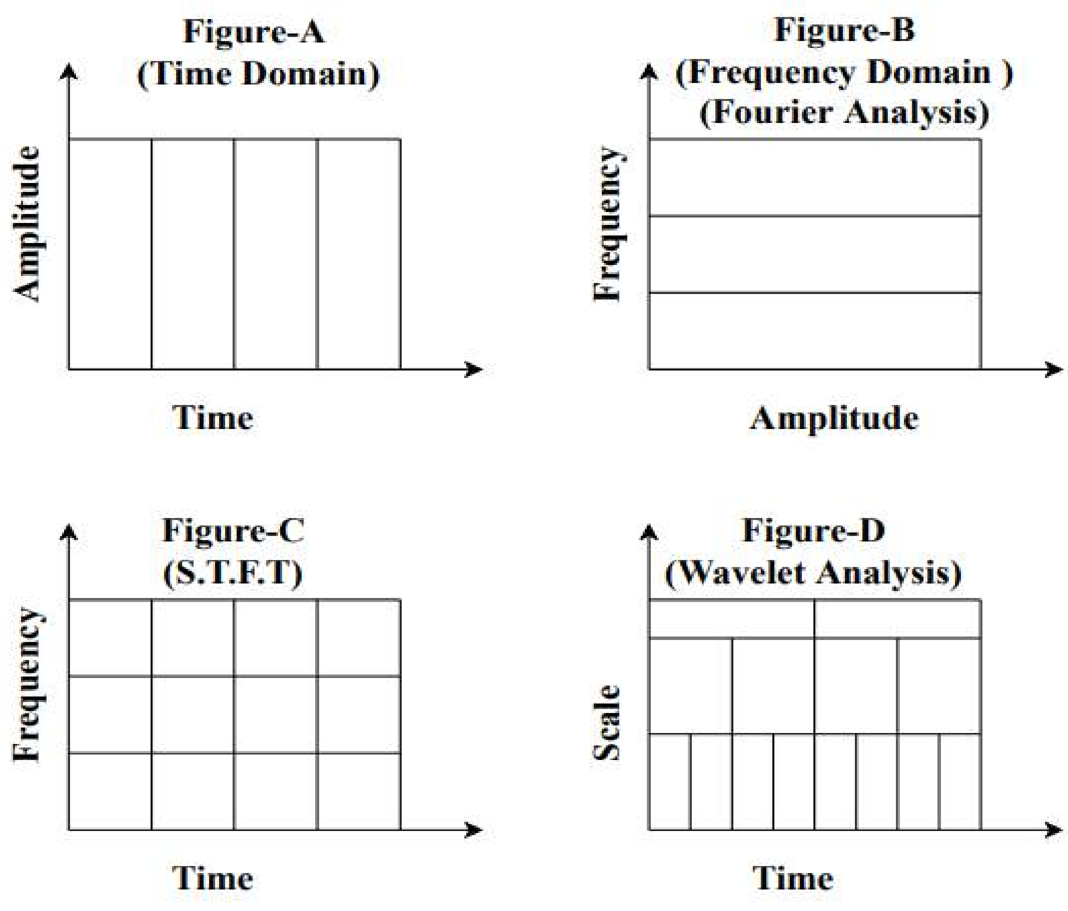

Fig.1.7**:** Representation of a Signal in Different Domains, i.e. "Time Domain, Frequency Domain, STFT and Wavelet Analysis".

From the above figures, figure-A gives time information but has zero information about frequency in the time domain, figure-B gives zero information about time, but gives good information about frequency in the frequency domain, figure-C anyway gives information about both time and frequency but the frequency information gained is dependent on the adopted window in the time domain. Because once the window size is fixed it remains the same along the time axis. Figure D gives information about both time and frequency by translating the wavelet by unfixed width throughout the time axis.

Finally, we can say that at a higher frequency, wavelet transform gives superior time resolution and bad frequency resolution, and at a lower frequency, it gives bad time resolution and good frequency resolution.

1.12.5 Wavelet

Wavelet may be defined as a mathematical function that displays the oscillatory fashion in a limited time. In other words, a wavelet is a small wave similar to oscillation with amplitude that starts increasing from zero and finally decreases back to zero. Example Morlet Wavelet;

Fig.1.8: Graph of Morlet Wavelet

This function is also called the bases function used for the wavelet transform. Symbolically it is denoted as $\psi(t)$

Mathematically a wavelet function is defined [27]as;

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right) \quad a,b \in R, a \neq 0 \qquad \text{............. (1.10)}$$

Here **a** is the scaling factor i.e., $f = \frac{1}{a}$ is a frequency, **b** is known as translation or window location on axis, and $\psi \in L^2(R)$ i.e., a set of orthogonal wavelet bases in $L^2(R)$.

This function is known as the mother wavelet. Graphically, we can differentiate wavelet function from wave function as;

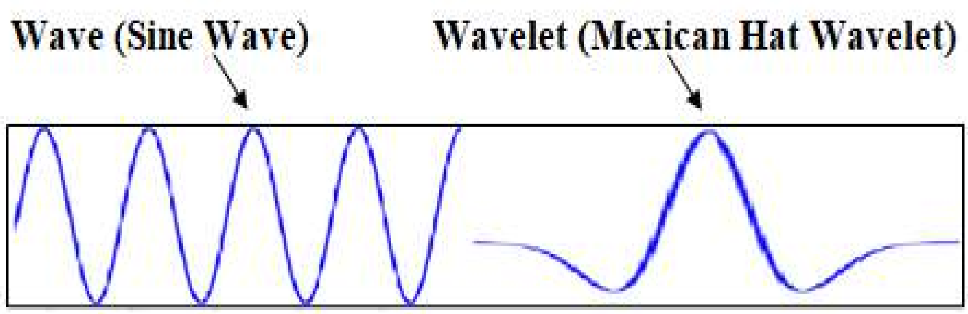

Fig.1.9: Graphical Representation of Wave and Wavelet

The wavelet function decomposed the input signal into several frequency components. It can generate a series of wavelet functions (daughter wavelets) of varied scales about the frequency or assign a frequency range to each scale component. These generated functions are small waves with a short duration. Then each divided signal component is analyzed with a resolution that matches its scale. In wavelet transformation two parameters play an important role, one is called the translation parameter and the second is the scaling parameter. Translation parameter means shifting the wavelet along the time axis, in order to find out the time information about a signal. While scaling parameter means changing the amplitude and time duration of the wavelet function in order to obtain frequency information.

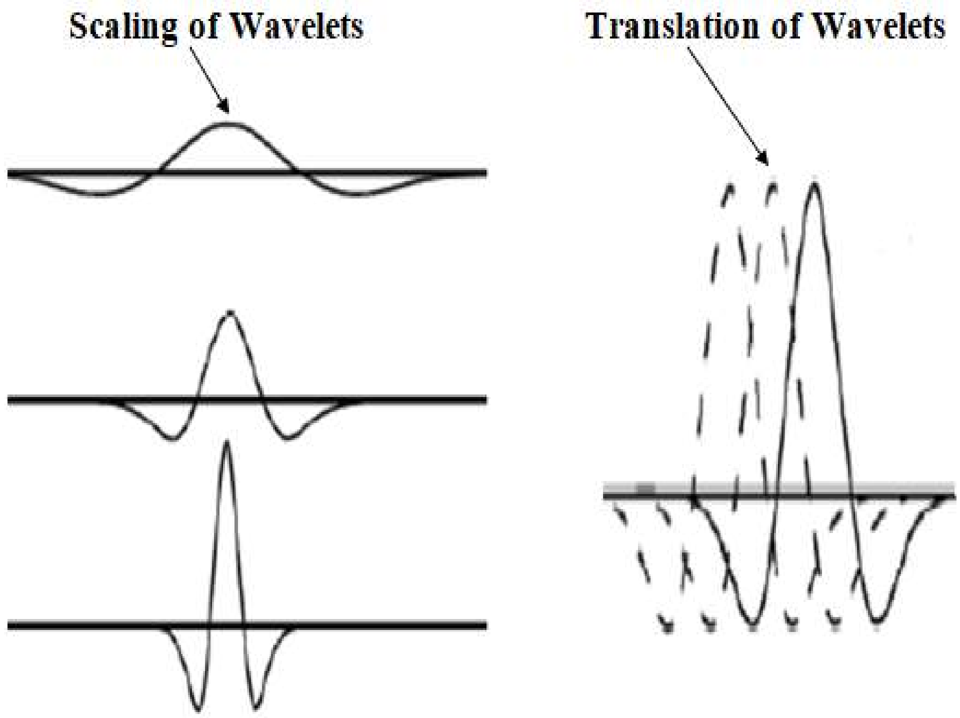

Fig.1.10: Scaling and Translation of a Wavelet

Any function is said to be a wavelet or mother wavelet if it satisfies certain conditions;

- Wavelet must have finite energy [27];

$$E = \int_{-\infty}^{+\infty} |\psi(t)|^2 \, dt < \infty \qquad \text{..............} \ (1.11)$$

  Here E is called the energy of the function, which is equal to the integral of its square magnitude.

- When $\psi(x)$ is the Fourier transform of $\psi(t)$ i.e.;

$$\psi(x) = \int_{-\infty}^{+\infty} \psi(t) e^{-i(2\pi x)t} dt \qquad \text{.............. (1.12)}$$

Then, the below condition must satisfied;

$$C_g = \int_{-\infty}^{\infty} \frac{|\psi(x)|^2}{x} dx < \infty \qquad \text{.............. (1.13)}$$

The above function is known as the admissibility condition. Here C_g is known as the admissibility constant. It means the wavelet has no zero frequency component or $\psi(0) = 0$ the wavelet $\psi(t)$ have zero mean[27].

Here, we are going to discuss several wavelets, used in this research work, as basis functions.

1.12.5.1 Haar wavelet

It is a simplest and discontinuous wavelet, mathematically defined as;

$$\psi(t) = \begin{cases} 1 \; ; & 0 \le t < 1/2 \\ -1 \; ; & 1/2 \le t < 1 \\ 0 \; ; & otherwise \end{cases} \qquad \text{............... (1.14)}$$

Here $\psi(t)$ is called Mother Wavelet

And, Its Father Wavelet $\phi(t)$ is defined as;

$$\phi(t) = \begin{cases} 1 \; ; & 0 \le t < 1 \\ 0 \; ; & elsewhere \end{cases} \qquad \text{............... (1.15)}$$

Here it's simple to observe that the function "$\psi(t-k):k\in N$" create an orthogonal set of bases function. It can be used for both discrete as well as continuous functions. A graphical representation of a Haar Wavelet and its dilation is given below;

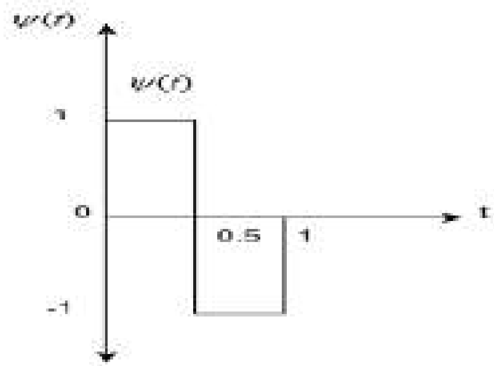

Fig.1.11: Graphical Representation of Haar Wavelet

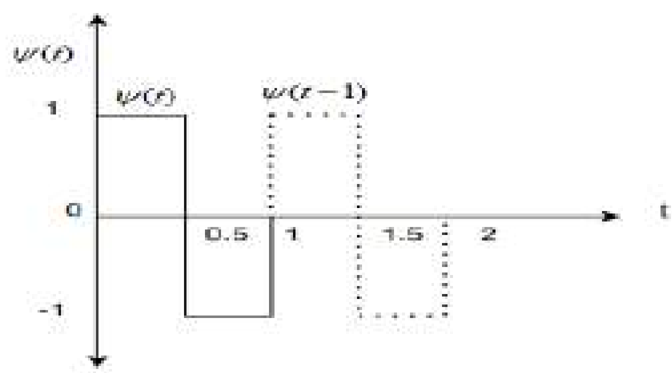

Fig.1.12: Graphical Representation of Dilation of Haar Wavelet

If we integrate, $\psi(t)$ and $\psi(t-1)$ together result equals to zero i.e.

$$\int_{-\infty}^{\infty}\psi(t)\psi(t-1)dt=0 \qquad \text{............... (1.16)}$$

Therefore, in general, we have;

$$\int_{-\infty}^{\infty}\psi(t-m)\psi(t-n)dt=\delta_{m-n} \qquad \text{............. (1.17)}$$

But, (1.17) equals to 1, when $m = n$, and 0 when m $m \neq n$ i.e.

$$\int_{-\infty}^{\infty} \psi(t)\psi(t)dt = 1 \quad \text{............... (1.18)}$$

1.12.5.2 Mexican Hat Wavelet (mexh)

Mexican Hat Wavelet is also called Richer Wavelet because it was originally introduced by American Geophysicist H. Richer. This wavelet function is a negative second derivative of a gaussian function. The orthogonal analysis is not possible, as it does not exist in scaling function. It is an even, real-valued and continuous type wavelet.[28].

Mathematically Mexican Hat Wavelet is denoted as[27];

$$\psi(t) = (1 - t^2)e^{\frac{-t^2}{2}} \quad \text{............... (1.19)}$$

Here $e^{\frac{-t^2}{2}}$ is gaussian function with unit variance.

It is graphically denoted as;

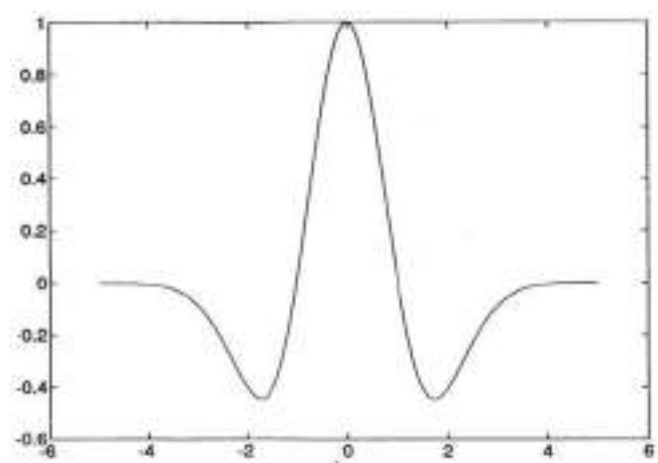

Fig.1.13: Graphical Representation of Mexican Wavelet

1.12.5.3 Daubechies Wavelet (dbN)

The dbN is a family of Daubechies wavelets and are, discrete, orthogonal, and compact supported orthogonal wavelets. The whole work of Daubechies was done on the bases of the theory and framework of Morlet and Grossmann and finally refined by Myers. These wavelets are applicable for different types of signals because it uses easy filtering thought. It cannot have an absolute expression, other than db1. The maximum present wavelets are asymmetric. Its wavelet function has N number of vanishing moments.

1.12.5.4 Orthonormal Wavelet

A wavelet function $\psi(t) \in L^2(R)$ is known as an orthogonal wavelet when a set of a wavelet function $\psi_{l,m}(t)\ \ l, m \in Z$, i.e.

$$\psi_{l,m}(t) = 2^{l/2}\psi\left(2^{l}t - m\right)$$ ....

(1.20**)**

is an "orthonormal base of the finite energy space $L^2(R)$". Here the orthogonality of the function is certain by the factor $2^{l/2}$.

1.12.5.5 Biorthogonal Wavelet (biro)

In a biorthogonal wavelet, the associated wavelet transform is invertible, but not necessarily orthogonal. It gives more degree of freedom than orthogonal wavelets i.e. the possibility to construct symmetric wavelets. In the biorthogonal type of function, we have two scaling functions i.e., $\phi\ and\ \phi$, which may help for multi-resolution analyses.

1.12.5.6 Meyer Wavelet (meyr)

Meyer wavelet was introduced by a famous mathematician Yves Meyer[28]. The name of the wavelet was chosen in his honor. He is also a part of the group of the founder of the wavelet theory. This wavelet is a continuous type and is applicable for several types of signal analyses. It is defined as;

$$\psi(t) = 2\int_{0}^{\infty} \sin(\omega(h)) \cos[2\pi(t - 1/2)h]dh \qquad \text{............... (1.21)}$$

The Meyer wavelet can be applied in several areas like "Ultrasonic Waves Processing, Audio & Speech Coding, Biomedical Signal Processing, Image Processing, Image Edge Detection, Signal Filtering and Image Compression" etc.

It can be graphically shown as;

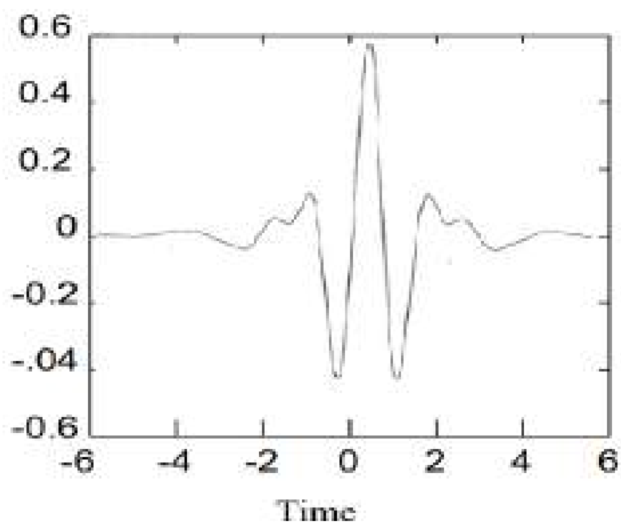

Fig.1.14: Graphical Representation of Meyer Wavelet

1.12.5.7 Morlet Wavelet (morl)

The Morlet Wavelet was introduced by French geophysicist Jean Morlet [28]**.** In 1975 he started working on wavelet theory. His wavelet is basically the combination of two functions, namely sinusoidal function and gaussian function. It has both-time as well as frequency localization features, are better. It is a continuous nature type wavelet.

Its mathematical function is given as;

$$\psi(x) = Ce^{-x^2/2}\cos(5x) \qquad \text{...............} (1.22)$$

Here constant C can be applied for the reconstruction of the signal.

Graphically Morlet Wavelet can be shown below as;

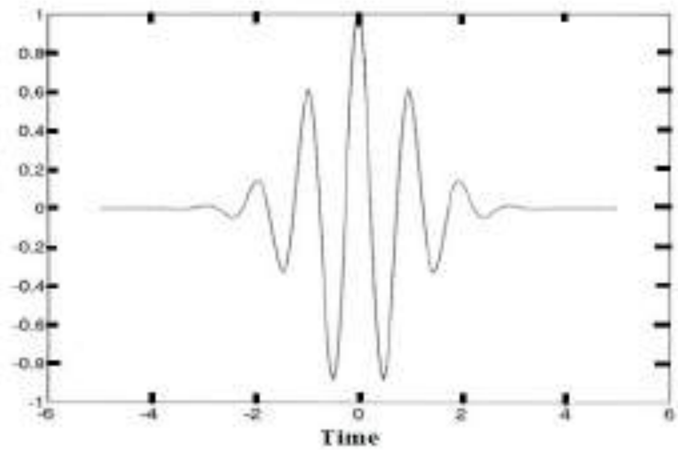

Fig.1.15: Graphical Representation of Morlet Wavelet

1.12.5.8 Symlet Wavelet (symN)

The Symlets wavelet was designed by Daubechies, by analysis and modification to the Daubechies set of wavelets. In order to make the properties of the two wavelet functions similar. This wavelet is close to an asymmetric wavelet. It is sometimes known as Daubechies "least asymmetric" wavelet. The set of wavelet are shown by SymN, N denotes the order. It is applied in several areas, like biomedical, communications, medicine, seismic, ultrasonic waves, audio speech coding, geophysics, filtering denoising, image, compression, etc.

1.12.5.9 Coiflet Wavelet (coif.)

The name of the wavelet coiflet[28] was given in honor of Ronald Coifman. Because on his request to Ingrid Daubechies to design a wavelet and to have scaling function with vanishing moments. These are orthogonal wavelets in which both the mother wavelet and the father wavelet have many vanishing moments after zero. It has $\frac{N}{3}$ vanishing moments and the connected scaling function has $\frac{N}{3}-1$ vanishing moments. It can be graphically shown as

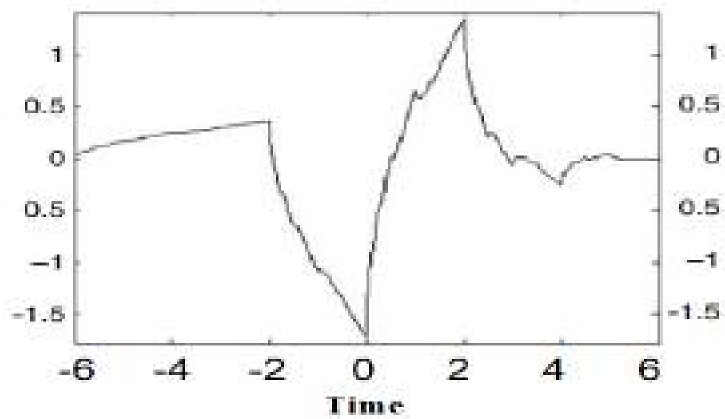

Fig.1.16: Graphical Representation of Coiflet Wavelet

1.12.6 Wavelet Transform

In wavelet transform, we require a mother wavelet, which should be more flexible than others existing before. We do two basic manipulations, in order to make wavelets more flexible, first we have to stretch and squeeze it, and second we have to shift it. Here dilation parameter is denoted by “a” and the translation parameter is denoted by “b”. Thus, if we put these parameters in the Mexicana Hat Wavelet, which is given above equation (1.19), it becomes[27];

$$\psi\left(\frac{t-b}{a}\right)=\left(1-\left(\frac{t-b}{a}\right)^{2}\right)e^{-\frac{1}{2}[(t-b)/a)]^{2}} \qquad \text{................ (1.23)}$$

It gives back to the original equation hat wavelet if we put a=1 and b=0 in equation (1.19). Thus in terms of the above equation (1.23), we will transform a function, $x(t)$ on the bases of ranges of a's and b's.

1.12.6.1 Continuous Wavelet Transform

The wavelet transform of signal $x(t)$ w.r.t wavelet function ψ is given as[27],[29];

$$T(a,b) = w(a)\int_{-\infty}^{\infty} x(t)\psi^*\left(\frac{t-b}{a}\right)dt \quad \text{...............} (1.24)$$

Here "a" is scale, "b" is translation, "$w(a)$" is a weighted function, "ψ" is wavelet function, "$*$" is complex conjugate ("applicable in case of complex function").

Therefore, we will get continuous wavelet transform (CWT), if simply we put $w(a) = \frac{1}{\sqrt{a}}$ in the above equation (1.24) i.e.

$$T(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} x(t)\psi^*\left(\frac{t-b}{a}\right)dt \quad \text{...............} (1.25)$$

Equation (1.25) it can be written as;

$$T(a,b) = \int_{-\infty}^{\infty} x(t)\psi^*_{a,b}(t)dt, \text{ or } T(a,b) = \langle x(t), \psi_{a,b}(t)\rangle \quad \text{...............}$$

(1.26)

Here $\frac{1}{\sqrt{a}}$ is the normalization factor "It indicates at every scale, wavelets have equal energy" Here integral indicates the product among two functions, namely analyzing signal and mother wavelet over signal length, and this process is also known as a convolution between $x(t)$, and ψ. The output result of CWT for a signal is in the shape of wavelet coefficients, they are actually the functions of scale and translation. The computing of the wavelet coefficient at “each scale and translation” by means of CWT can produces huge data. So CWT takes much more time for calculating data about any signal. To avoid such calculations, we have to adopt a process called a “dyadic scaling and shifting”.

1.12.6.2 Discrete Wavelet Transform

As we have Discrete Fourier Transform (D.FT) and Discrete Short Time Fourier Transform (D.STFT), in the same fashion we have Discrete Wavelet Transform (DWT). As in Fourier theory we discretize time and frequency axes, but in DWT we extract the discrete values from the scale and translation parameters by a distinct manner. Here is a motive to introduce DWT, and then explain connection between DWT and IWT[25],[28].

Now, we will give another shape of scale parameter and translation parameter below as;

Let $a = 2^{-l}$ *and* $b = k2^{-l},\ k,l \in Z$

Put these values in equation (1.25), we get

$$T(2^{-l}, k2^{-l}) = 2^{l/2} \int_{-\infty}^{\infty} x(t)\psi\left(2^{l}t - k\right)dt \qquad \text{............... (1.27)}$$

The above applied wavelet is also known as a dyadic grid wavelet i.e.;

$$\psi_{l,k}(t) = 2^{l/2}\psi(2^{l}t - k) \qquad \text{..............}$$

(1.28)

1.12.7 Wavelet Packet Transform

The Wavelet Packet Transform (WPT) was introduced by Meyer Coifman and other researchers. In reality, it is an extension of wavelet and multiresolution theory. The wavelet transform is universal to create a set of the orthogonal basis of wavelet packets. The wavelet packet has strength and power for deciding which bases function is perfect to represent the given signal. It has much more flexibility for using the basis function for the frequency contents of a given signal. The WPF's are generated by scaling and translation of a set of bases functions, they may be mother and father wavelets i.e. ψ and ϕ [30].

In WPT both types of coefficients (approximation as well as detailed coefficients) are again decomposed at each level for approximations and details parts, which is not possible in WT. In Wavelet transformation there is a chance of discard of information because after the first level of decomposition only approximation coefficient is further decomposed. But in wavelet packet transform both approximations as well as detailed coefficients are further decomposed at each level. The process of analyzing a given signal by WPT is shown below;

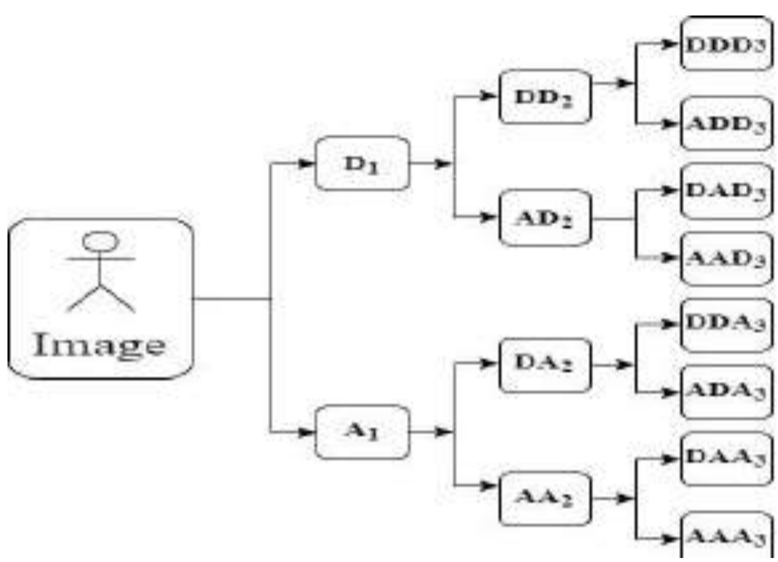

Fig.1. 17: Graphical Representation of WPT of a given Signal.

1.12.8 Inverse Wavelet Transform(IWT)

In simple, inverse wavelet transformations (IWT) mean the reconstruction of the signal, which were under wavelet transformation. In other words, we decompose a signal by wavelet transformation for analysis purposes, then we need a transformation to get (or reconstruct) the signal back, which is known as inverse wavelet transform.

Mathematically it is written as;

$$f(t)=\frac{1}{C\psi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}W_{\psi}f(a,b)\psi_{a,b}(t)(\frac{1}{a^2}da)db$$
$$or\ \ f(t)=\frac{1}{C\psi}\int_{0}^{\infty}\ \frac{1}{a^2}[W_{\psi}f(a,b)]\psi_{a,b}(t)da\int_{-\infty}^{\infty}\ db \qquad \text{............... (1.34)}$$

here, $C_{\psi}=\int_{-\infty}^{\infty}\left|\frac{\psi(\omega)}{\omega}\right|d\omega<0$ is known as the admissibility condition**.** In digital image processing, the IWT is applied to synthesizing the input signal. The IWT provides us the synthesized result about the signal which is analyzed by WT. Sometimes we say it is a reverse process of WT under certain conditions.

1.12.9 Multiresolution (MRA)Analysis

Multiresolution simply means analyses of every component of a signal at different scales (different window sizes). In other words, by applying the MRA, we can break a difficult function into a number of simpler parts and then study them individually. One of the important purposes of these analyses is to obtain an approximation of function $f(x)$ at different resolutions. MRA was introduced by Yves Mayer and Mallat in 1989. It is basically applied for constructing an orthogonal wavelet basis for the space L^2. It is a set of the closed subspace of L^2 and satisfying certain conditions[25],[31]

1.13 Objectives of Research

- Importance of Wavelet Transform in Digital Image Processing, especially Denoising of Digital Grayscale Images.
- Denoising of Digital Images through various Filters and Wavelet Functions.
- Denoising of Digital Image applying different Thresholding Parameters and Wavelet Transform.
- Performance comparison of various wavelet functions for denoising of digital images based on SNR and Wavelet Transform.
- Importance of Norm in digital image denoising.

1.14 Thesis Structure

The outlines of the various chapters of this thesis are as follows:

- **Chapter 1:** Introduction.
- **Chapter 2:** Digital Image denoising by different Filters and Wavelet Transformation.
- **Chapter 3:** Digital Image Denoising by a Thresholding Technique and Wavelet Packet Transformation.
- **Chapter 4:** Estimated Threshold for Digital Image Denoising Through Wavelet Packet Transformation.
- **Chapter 5:** Importance of Norm in a Digital Image denoising.

1.14 A sample programs for image processing in *MATLAB*.

%% 2-D-discrete wavelet transform + IDWT) For-true color-image%%

```
%Read Input Image%;
Input_Image=imread('barbara download.jpeg');
%Red Component of Colour Image
Red_Input_Image=Input_Image(:,:,1);
%Green Component of Colour Image
Green_Input_Image=Input_Image(:,:,2);
%Blue Component of Colour Image
Blue_Input_Image=Input_Image(:,:,3);
%Apply Two Dimensional Discrete Wavelet Transform
[LLr,LHr,HLr,HHr]=dwt2(Red_Input_Image,'haar');
[LLg,LHg,HLg,HHg]=dwt2(Green_Input_Image,'haar');
[LLb,LHb,HLb,HHb]=dwt2(Blue_Input_Image,'haar');
%Apply 2-D Inverse Discrete Wavelet Transform (idwt)
First_Level_Decomposition(:,:,1)=idwt2(LLr,LHr,HLr,HHr,'haar',size(Input
_Image));
First_Level_Decomposition(:,:,2)=idwt2(LLg,LHg,HLg,HHg,'haar',size(Input
_Image));
First_Level_Decomposition(:,:,3)=idwt2(LLb,LHb,HLb,HHb,'haar',size(Input
_Image));
First_Level_Decomposition=uint8(First_Level_Decomposition);
%Display Image
figure;
subplot(1,2,1);imshow(Input_Image);title('Input Image');
subplot(1,2,2);imshow(First_Level_Decomposition,[]);title('Reconstructed
image');
```

%%Wavelet Packet Transform For Image Denoising%%

```
%%[filename,pathname]=uigetfile('GRAYSCALE IMAGE.jpg');
%%filewithpath=strcat(pathname,filename);
img=imread('TEST IMAGE OF BARABRA .jpg');
imshow(img);
imgn=imnoise(img,'gaussian',0,0.01); %%add noise
wname='haar';level=3;keepap=1;
crit='shannon';critv=0;
sorh='s';
dwtmode('per');%signal Extension Mode;
wpt=wpdec2(double(imgn),level,wname,crit,critv);   %% WPD Decomposition
det1=[wpcoef(wpt,2) wpcoef(wpt,3) wpcoef(wpt,4)]; %%collecting detailed
coefficients
sigma=20 %% Estimation noise variance
T=sigma*sqrt(2*log(length(det1(:))));%% To find threshold
%% T=wthmngr('wp2ddenoGBL','sqtwologswn',wpt);%%find threshold
thrwp=T/3 %%Threshold adjust
imgd=wpdencmp(wpt,sorh,'nobest',thrwp,keepap);%%Denoising with full tree
%%imgd=wpdencmp(wpt,sorh,crit,thrwp,keepap);%% Denoising with best tree
imgd8=uint8(imgd);
%% Calculate MSE,SNR,PSNR
[Mc,Nc]=size(img);
MSE=sum(sum(((imgd-double(img)).^2 ))/(Mc*Nc));
maxp=max(max(double(imgd(:)),max(double(img(:)))));
PSNR=10*log10((maxp^2)/MSE);
SNR=20*log10(norm(double(img(:)))/norm(double(img(:))-double(imgd(:))));
subplot(2,2,1)
imshow(img),title('Input image')
subplot(2,2,2);
imshow(imgn);title('Noisy Image');
subplot(2,2,3);
imshow(imgd8);title('Denoised Image');
```

%%Digital image decomposition by wavelet theory at certain levels%%

```
%% Daubechies Wavelet Transform (db2) for 2-D signal (RGB Digital Images)
clear
Clear all;
x=imread ('Test RGB digital images')%% Load input RGB digital image
Imshow(x); subplot (1,1,1);%% Show loaded digital image
%%Apply Two Dimensional Discrete Wavelet Transform at level-1
[xar,xhr, xvr,xdr]=dwt2(x(:,:,1),'db2');%%red color cannel
[xag,xhg,xvg,xdg]=dwt2(x(:,:,2),'db2');%%green color cannel
[xab,xhb,xvb,xdb]=dwt2(x(:,:,3),'db2');%% blue color cannel
xa(:,:,1)=xar; xa(:,:,2)=xag; xa(:,:,3)=xab;%% Approximation coefficients
xh(:,:,1)=xhr; xh(:,:,2)=xhg; xh(:,:,3)=xhb;%% Horizontal coefficients
xv(:,:,1)=xvr; xv(:,:,2)=xvg; xv(:,:,3)=xvb;%% Vertical coefficients
xd(:,:,1)=xdr; xd(:,:,2)=xdg; xd(:,:,3)=xdb;%% Diagonal coefficients
figure,imshow(xa/100); Approximation image
figure,imshow(xh);%% Horizontal Image
figure,imshow(xv);%% Vertical Image
figure,imshow(xd);%% Diagonal Image
x1=[xa*0.03 log10(xv)*0.3 ; log10(xh)*0.3 log10(xd)*0.3];
figure,imshow(x1);%%First level decomposition level of a digital image
%Apply Two Dimensional Discrete Wavelet Transform at level-2
[xaar,xhhr,xvvr,xddr]=dwt2(x(:,:,1),'db2');
[xaag,xhhg,xvvg,xddg]=dwt2(x(:,:,2),'db2');
[xaab,xhhb,xvvb,xddb]=dwt2(x(:,:,3),'db2');
xaa(:,:,1)=xaar; xaa(:,:,2)=xag; xaa(:,:,3)=xaab;
xhh(:,:,1)=xhhr; xhh(:,:,2)=xhg; xhh(:,:,3)=xhhb;
xvv(:,:,1)=xvvr; xvv(:,:,2)=xvg; xvv(:,:,3)=xvvb;
xdd(:,:,1)=xddr; xdd(:,:,2)=xdg; xdd(:,:,3)=xddb;
figure,imshow(xaa/100);
figure,imshow(xhh);
figure,imshow(xvv);
figure,imshow(xdd);
x11=[xaa*0.03 log10(xvv)*0.03 ; log10(xhh)*0.3 log10(xdd)*0.3];
figure;imshow(x11);
%Apply Two Dimensional Discrete Wavelet Transform at level-3
[xaaar,xhhhr,xvvvr,xdddr]=dwt2(x(:,:,1),'db2');
[xaaag,xhhhg,xvvvg,xdddg]=dwt2(x(:,:,2),'db2');
[xaaab,xhhhb,xvvvb,xdddb]=dwt2(x(:,:,3),'db2');
xaaa(:,:,1)=xaaar; xaaa(:,:,2)=xag; xaaa(:,:,3)=xaaab;
xhhh(:,:,1)=xhhhr; xhhh(:,:,2)=xhg; xhhh(:,:,3)=xhhhb;
xvvv(:,:,1)=xvvvr; xvvv(:,:,2)=xvg; xvvv(:,:,3)=xvvvb;
xddd(:,:,1)=xdddr; xddd(:,:,2)=xdg; xddd(:,:,3)=xdddb;
figure,imshow(xaaa/100);
figure,imshow(xhhh);
figure,imshow(xvvv);
figure,imshow(xddd);
x111=[xaaa*0.03 log10(xvvv)*0.03 ; log10(xhhh)*0.3 log10(xddd)*0.3];
figure;imshow(x111);
%% Apply Two Dimensional Inverse Discrete Wavelet Transform
First_Level_Decomposition(:,:,1)=idwt2(xaaar,xhhhr,xvvvr,xdddr,'db2',size(x111));
First_Level_Decomposition(:,:,2)=idwt2(xaaag,xhhhg,xvvvg,xdddg,'db2',size(x111));
First_Level_Decomposition(:,:,3)=idwt2(xaaab,xhhhb,xvvvb,xdddb,'db2',size(x111));
First_Level_Decomposition=uint8(First_Level_Decomposition);
%Display Image
imshow(First_Level_Decomposition); figure;
subplot(1,2,1); imshow(x); title('My Orgional image');
subplot(1,2,2); imshow(First_Level_Decomposition,[]); title('Reconstructed image');
```

%%Lena digital Image denoising%%

```
%%[filename,pathname]=uigetfile('GRAYSCALE IMAGE.jpg');
%%filewithpath=strcat(pathname,filename);
img=imread('Lena gray scale image for new paper.jpg');
imshow(img);
imgn=imnoise(img,'gaussian',0,0.01); %%add noise
wname='sym4';level=3;keepap=1;
crit='shannon';critv=0;
sorh='s';
dwtmode('per');%signal Extension Mode;
wpt=wpdec2(double(imgn),level,wname,crit,critv);  %% WPD Decomposition
det1=[wpcoef(wpt,2) wpcoef(wpt,3) wpcoef(wpt,4)]; %%collecting detailed
coefficients
sigma=median(abs(det1(:)))/0.6745; %% Estimation noise variance
T=sigma*sqrt(2*log(length(det1(:))));%% To find threshold
%% T=wthmngr('wp2ddenoGBL','sqtwologswn',wpt);%%find threshold
thrwp=T; %%Threshold adjust
imgd=wpdencmp(wpt,sorh,'nobest',thrwp,keepap);%%Denoising with full tree
%%imgd=wpdencmp(wpt,sorh,crit,thrwp,keepap);%% Denoising with best tree
imgd8=uint8(imgd);
img8=uint8(img);
%% Calculate PSNR
[Mc,Nc]=size(img);
MSE=sum(sum((imgd-double(img)).^2))/(Mc*Nc);
maxp=max(max(imgd(:)),max(double(img(:))));
PSNR=10*log10((maxp^2)/MSE);
SNR=20*log10((sum(sum((((imgd).^2)./((img) -double(imgd)).^2))))));
fprintf('\n,Orgional_vs_Denoised_PSNR = % f
\n',Original_vs_Denoised_PSNR);
imt=uint8(255*ones([Mc,Nc]));
PSNR=num2str(Original_vs_Denoised_PSNR);
imtp=insertText(imt,[Mc/4,Nc/2],PSNR,'FontSize',30);
subplot(2,2,1)
imshow(img),title('Input image')
subplot(2,2,2);
imshow(imgn);title('Noisy Image');
subplot(2,2,3);
imshow(imgd8);title('Denoised Image');
subplot(2,2,4);
imshow(imtp);title('PSNR');
```

%%Image denoising and Norm Calculation%%

```
%%Img:input noisy image image
%%Programme
%% load original image
img=imread('Barbara 256?256.jpeg');
%% Add Noise (Gaussian White Noise (0.003)%%
imgn=imnoise(img,'gaussian',0,0.003);%% gaussian noise with zero mean
and
dwtmode('per');%% single extension mode
[thr,sorh,keepapp]=ddencmp ('den','wv',imgn); % finding default value
%Denoising noisy image by global thresholding
imgden=wdencmp('gbl',double(imgn),'stm4',1,thr,sorh,keepapp);
%(3xN) Threshold matrix for 'lvd' option.
%thr=[thr,thr;thr thr;thr thr];%denoise image using level dependent
thresholding
% imag=wdencmp('lvd',double(imgn)),'sym4',5,thr,'s');
%PLOTTING
subplot(131);imshow(img),title('Original Image');
subplot(132);imshow(imgn),title('Noisy Image');
subplot(133);imshow(uint8(imgden),[]), title('Denoised  Image');
%%Finding SNR and Norms
Orgional_vs_Noisy_SNR=20*log10(norm(double(img(:)))/norm(double(img(:))-
double(imgn(:))));
Orionalg_vs_denoisd_SNR=20*log10(norm(double(imgden(:)))/norm(double(img
(:))-double(imgden(:))));
L1=max(sum(abs(uint8(img)-uint8(imgden))));
L2=max(sqrt(sum(abs(uint8(img)-uint8(imgden)).^2)));
```

%%Global thresholding%%

```
clc;
clear all;
warning off;
x=imread('GRAYSCALE IMAGE.jpg');
imshow(x);
title ('Orgional Image');
figure;
d=im2double(x);
imshow(d);
title('Global Thresholding using Matlab');
Id=x %I is a unit8 gray scale Image;
T=0.5*(min(min(Id)+max(Id(:))));
deltaT=.0001;%CONVERGENCE CREATION
done=false;
while~done
    g=Id>=T;
    Tnext=0.5*(mean(Id(g))+mean (Id(~g)));
    T=Tnext
end
figure;
d=im2w(x,T);
imshow(d);
title('Global Thresholding using our own code');
```

%%DWT +NOISE+THRESHOLD+IDWT FOR 2-D Images%%

```
clear all,
I=imread (uint8('POUT.jpg'));
J=imnoise(I,'gaussian',0.005);
subplot(1,2,1); imshow(I); title('Original image');
subplot(1,2,2); imshow(J); title('Noise Added Image');
%apply discrete wavelet transform to noisy image%
type='h';
[cA1,cH1,cV1,cD1]=dwt2(J,'haar');
[cA2,cH2,cV2,cD2]=dwt2(cA1,'haar');
[cA3,cH3,cV3,cD3]=dwt2(cA2,'haar');
%% sigthresh = threshold to remove outliers (default = 3)
T_cH3=sigthresh(cH3,3,cH3);
T_cV3=sigthresh(cH3,3,cV3);
T_cD3=sigthresh(cH3,3,cD3);

Y_cH3=wthresh(cH3,type,T_cH3);
Y_cV3=wthresh(cV3,type,T_cV3);
Y_cD3=wthresh(cD3,type,T_cD3);

T_cH2=sigthresh(cH2,2,cH2);
T_cV2=sigthresh(cH2,2,cV2);
T_cD2=sigthresh(cH2,2,cD2);

Y_cH2=wthresh(cH2,type,T_cH2);
Y_cV2=wthresh(cV2,type,T_cV2);
Y_cD2=wthresh(cD2,type,T_cH2);

T_cH1=sigthresh(cH1,1,cH1);
T_cV1=sigthresh(cH1,1,cV1);
T_cD1=sigthresh(cH1,1,cD1);

Y_cH1=wthresh(cH1,type,T_cH1);
Y_cV1=wthresh(cV1,type,T_cV1);
Y_cH1=wthresh(cD1,type,T_c1);

%INVERSE DSICRETE WAVELET TRANSFORM (idwt)
Y_cA2=idwt2(cA3,Y_cH3,Y_cV3,Y_cD3,'haar');
Y_cA1=idwt2(cA2,Y_cH32,Y_cV2,Y_cD2,'haar');

Y_j=idwt2(Y_cA1,Y_cH1,Y_cV1,Y_cD1,'haar');

%Find PSNR
error_diff=I-unit(Y_j);

%PSNR = 20*log10(255^2/err.^2);
```

%% Speckle noise + wame (sym4) For digital image denoising%%

```
%% speckle noise + wame (sym4)
img=rgb2gray(imread('CT-Scane Chest.jpg image.jpg'));
[rows columns] = size(img);
%% add noise%%
imgn=imnoise(img,'speckle',0.002);%% gaussian noise with zero mean and
noise variance 0.005
   dwtmode('per');%% single extension mode
[thr,sorh,keepapp]=ddencmp ('den','wv',imgn); % finding default value
%Denoising noisy image by global thresholding
imgden=wdencmp('gbl',double(imgn),'sym4',2,thr,sorh,keepapp);
%(3xN) Threshold matrix for 'lvd' option.
%thr=[thr,thr;thr thr;thr thr];%denoise image using level dependent
thresholding
% imag=wdencmp('lvd',double(imgn)),'sym4',2,thr,'s');
%PLOTTING
subplot(131);imshow(img),title('Orgional Image');
subplot(132);imshow(imgn),title('Noisy Image');
subplot(133);imshow(uint8(imgden),[]), title('denoised Image');
%%Finding MSE,PANR and SNR
%%Finding MSE,PANR and SNR
MSE= sum(sum((double(img) - double(imgden)).^ 2))/(rows * columns);
PSNR = 10 * log10( 256^2/MSE)
SNR=20*log10(norm(double(img(:)))/norm(double(img(:))-
double(imgden(:))));
essage = sprintf('The mean square error is %.2f.\nThe PSNR = %.2f', MSE,
PSNR);
```

Chapter 2

DIGITAL IMAGE DENOISING BY DIFFERENT FILTERS AND WAVELET TRANSFORMATION

2.1 Introduction

In the modern world, digital images are considered a wonderful source of information for various purposes. In the first chapter of this thesis, we mentioned different types of digital images. In modern medical sciences, medical imaging plays an important role in the detection of several types of diseases. Different types of machinery are available in the market for human body disease examinations. In medical machines, different types of cameras are fitted to capture images or videos (set of images) of the human body internally and externally for disease spot identification purposes.

According to the different recent research articles, for medical image analysis purposes, we selected a grayscale CT– Scan image (255×255) of the human chest as a test image. The CT- Scan facility for human purposes was applied as a clinical application in 1971. It is an enhanced version of the X-ray imaging technology that is restricted to imaging the body. This CT-scan images can be disturbed and affected due to certain reasons.

The CT images may be contaminated by gaussian noise and may decrease the clarity of low-contrast objects. Sometimes CT-scan images may corrupt due to gaussian noise, because of the presence of the electrical signals. It may also be contaminated by artifacts and structural noise. To view the objects in CT-scan image, the number of times at "high-speed computation creates "thermal energy fluctuation" and becomes the cause of noise entrance in the images. It may be affected by the noise produced by "Mathematical

Computational and Quantum Statistics”. Therefore, noise becomes a hurdle in obtaining correct information about the original digital images. Noisy signals (or unwanted signals) may corrupt the original signal fully or partially. The contaminated CT-scan image (or noisy CT-scan image) sometimes create problems for medical doctors in distinguishing among tissues of different densities.

Therefore, the removal of noise from the affected images becomes an essential part of modern research. Different techniques and methods have been applied to reduce the noise from noisy images. Optimal Weight Method (OWM)[32]. Selective Mean Filter(SMF)[33]. Blind Image De-convolution (BID) method[11].Wavelet Domain Filtering (WDF)method. Non Local Mean (NLM) and wavelet packet-based thresholding[34].

2.2 Digital Image Filtering Techniques

There are several filtering techniques used in image processing. Here we mention some common filtering techniques used in digital image enhancement, especially for digital image noise reduction[26].

2.2.1 Gaussian Filter

Gaussian filter is used to reduce the noise and the unnecessary details of the digital images. It is often used to blur digital images. It gives small weight to the pixels further from the center of the window[35].

2.2.2 Median Filter

The median filter is another best approach for noise removal in digital images, without any smoothing effects**.** It gives better results in terms of noise removal when added impulse noise like salt & pepper noise[26],[35].

2.2.3 Wiener Filter

The winner filter is a type of additive filter. It performs small smoothing when the variance is large and the other hand performs large smoothing when the variance is small and vice-versa. The winner filter is more selective than the other common linear filters because it preserves digital image edges as well as the high-frequency parts of an image[26],[35].

2.2.4 Guided Filter

It is called a guided filter because it uses the contents of the other images to perform edge preservation of the digital images. The image under consideration for guidance may be the image itself. But if both images are the same then their structures are the same and if they are different, the structure in the guidance image effect the filtered image**.**

2.2.5 Block Matching and 3-D Filtering (BM3-D)

This is a superior and difficult technique for image filtering. It performs its operation by grouping similar two- dimensional image fragments into a three-dimensional data array, known as “groups”. In this filtering approach, the good details are shown by grouped blocks and protect the important features of each block.

2.2.6 Additive Fuzzy Switching Median Filter

This type of filtering approach is also known as a noise adaptive filter. For the filtering process of the images, it first uses the histogram of the corrupted images, so that to identify noisy pixels. Then these identified noisy pixels are utilized for filtering action, here denoised pixels are retained and left unprocessed. Finally, for the filtering process, the filters utilize "fuzzy logic to handle uncertainty present in the external local information".

2.2.7 Linear Filter

In image processing, linear filtering is a unique technique applied for noise reduction, sharpening the image edges, and correcting unequal illumination. It is called a linear filter because the value of the output image pixel is a linear combination of the value of the pixel in the input pixels neighbourhood. The filtering technique is done by filtering the input image with the correlation of an appropriate filter kernel[26],[35].

2.2.8 Min Filter

This filtering technique is used to find out the darkest points in the digital images, and then enhance the dark area of the digital image. It's also used to find minimum values in the area surrounded by the filter. It's sometimes used to remove the impulse noise like salt noise, on the of minimum operations.

2.2.9 Max Filter

This filtering technique is used to find out the brightest points in the digital images, and then enhance the bright area of the digital image. It's also used to find maximum values in the area surrounded by the filter. It's sometimes used to remove the impulse noise like pepper noise, because of max operations

2.3 Wavelet Decomposition (DWT) of a Digital Image

The Wavelet Decomposition refers to the decomposition of the signal into varied frequency sub-bands[36][35]. These sub-bands are like "LL_i, LH_k, HL_k and HH_k", here $k = 1, 2, ...i$ or k^{th} frequency level, i represents the highest scale in the decomposition. Here LL_i is the lowest frequency sub-band called the Approximation Coefficient of the analyzing signal, and LH_k, HL_k and HH_k are known as the "Horizontal Coefficient, Vertical Coefficients, and Diagonal Coefficients" respectively. At the level first, a signal is decomposed into four parts i.e. "Approximation Coefficients part, Horizontal Coefficients part, Vertical Coefficients part, and Diagonal Coefficients part". But at level 2nd only the Approximation part is further decomposed. Because the Approximation coefficients were passed by low pass filters. In this way, the whole process of signal decomposition is done. The wavelet decomposition can be shown by below diagrams[36];

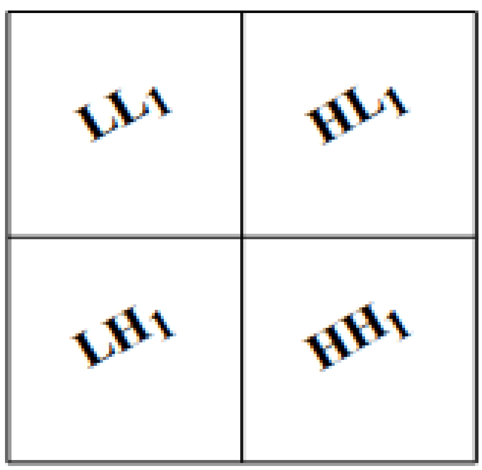

Fig.2.1:1st Level of DWT

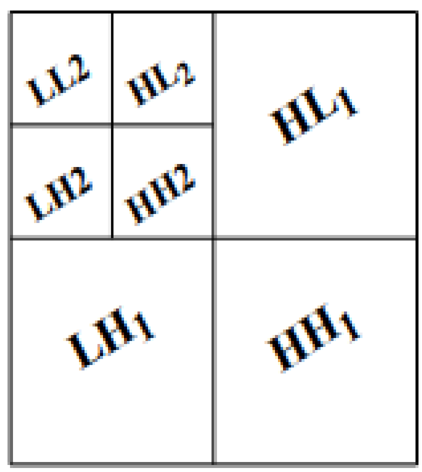

Fig.2.2: 2nd Level DWT

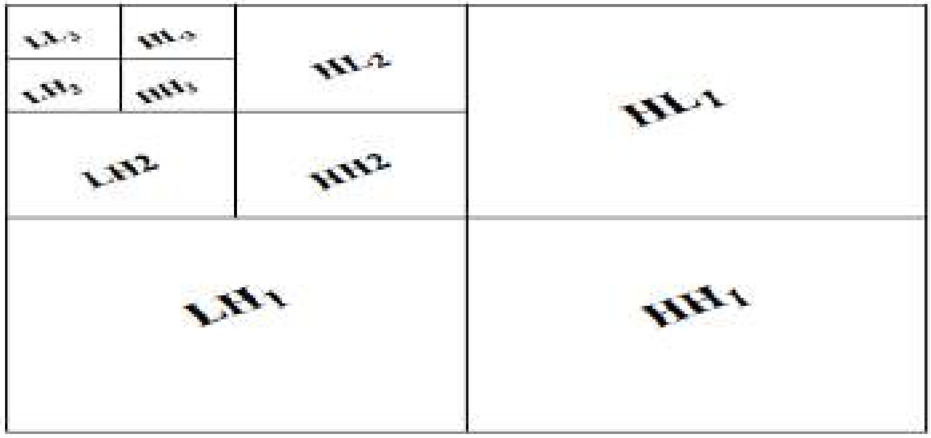

Fig.2.3: 3rd Level DWT

2.4 Variance

In Mathematical terms, we define variance as[36];

$$\textbf{Variance } (\sigma^2) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\frac{(f(x,y)-\overline{f}(x,y)}{M \times N} \qquad \text{………… (2.1)}$$

Here, $f(x,y)$ is an estimated function, $\overline{f}(x,y)$ is the mean of the estimated function, and σ^2 is a difference of "predicted values from its mean"

2.5 Signal Noise Ratio

Mathematically we write the SNR function as;

$$\text{SNR}= 20 \times \log_{10} \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\hat{f}(\text{x, y})]^2 \Big/ \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f(\text{x, y}) - \hat{f}(\text{x, y})]^2 \qquad \text{………… (2.2)}$$

Here, $f(\text{x, y})$ is the input image and $\hat{f}(\text{x, y})$ is synthesized image

2.6 Mean Square Error

The MSE function is defined as[36];

$$\text{MSE} = \frac{1}{M \times N}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f(\text{x, y}) - \hat{f}(\text{x, y})]^2 \qquad \text{………… (2.3)}$$

As above mentioned, $f(\mathrm{x,y})$ is an input image and $\hat{f}(\mathrm{x,y})$ is synthesized image

2.7 Peak Signal Noise Ratio

The mathematical form of a PSNR is given as[36];

$$\mathrm{PSNR} = 10 \times \log_{10} \frac{255^2}{\frac{1}{M \times N} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(\mathrm{x,y}) - \hat{f}(\mathrm{x,y})]^2} \text{ or } 10 \times \log_{10} \frac{255^2}{MSE} \qquad \ldots\ldots\ldots\ldots (2.4)$$

Here, 255×255 is the size of the input image, $f(\mathrm{x,y})$ the input image, and $\hat{f}(\mathrm{x,y})$ is synthesized image.

2.8 Thresholding Technique and Its Types

2.8.1 Thresholding

Thresholding is a technique of image segmentation. In other words, it decomposes or divides the image into different parts or sections. The decomposition process involves the partitioning of the image into a region corresponding to objects. We are doing segments of the regions to recognize common features. One of the important qualities of pixels in a region is that they may share intensity. Therefore thresholding is a unique technique to divide these regions. Sometimes it is used to divide the light and dark regions[36].

This technique is also used to produce binary images from grayscale images, by converting all the pixel values above than threshold to one and all the pixel values less than the threshold to zero.

When, $h(x,y)$ is the threshold of $I(x,y)$ at a global threshold T, then mathematically can be shown as;

$$h(x,y)=\begin{cases}1 & if\ I(x,y)\geq T\\ 0 & otherwise\end{cases} \qquad \ldots\ldots\ldots\ldots.. (2.5)$$

Here $h(x,y)$ is the threshold at (x, y) and $I(x,y)$ a gray-scale image at (x, y).

In digital image processing, the thresholding technique plays an important role in image denoising. In other words, the thresholding approach removes the unnecessary and unwanted features from the given images and also helps in showing hidden information in the images. Wavelet thresholding is also known as wavelet shrinkage. In the wavelet thresholding technique, each small coefficient gained from wavelet decomposition is decreased to zero. Usually, these big wavelet coefficients represent the real image and the small wavelet coefficients represent the noisy image. From time to time, different thresholding techniques were proposed for digital image denoising. But the most common and broadly applied thresholding techniques are hard and soft thresholding, introduced by Donoho in 1994.

This technique helps in the removal of the wavelet coefficient which is taking the noise beneath our selected threshold value and the rest coefficients could perform the requirement for estimation of the signal.

Therefore, to remove the noise from noisy images, different approaches and techniques are needed. In this research work, we used different methods and techniques for digital image denoising. Among these techniques, wavelet thresholding is one of the important techniques used for digital image denoising. Thresholding technique includes Hard Thresholding, Soft Thresholding, Hybrid Thresholding, BayesShrink Thresholding, VisuShrink thresholding, SureShrink Thresholding, etc. In the application of thresholding for digital image denoising, it equates small wavelet coefficients to zero, because these are the noisy wavelet coefficients present in the image. The theory of thresholding is based on threshold value T. Letting its value small, there may remain a small amount of wavelet coefficients that carry unwanted signal and these wavelets remain in the predictable signal. But still, our image remained noisy. On the other hand, if we choose a big value T it may eliminate some noise-free wavelet coefficient and it may create the problem of losing information. Again it becomes an obstacle in the way of digital image denoising. This type of problem can be indicated by SNR and PSNR. In another word, if SNR and PSNR are higher, superior is the accuracy of the digital image denoising. Finally, we conclude by saying that, we must choose an optimal threshold to get higher SNR and PSNR.

2.8.2 Types of Thresholding

The two general ways of thresholding the wavelet coefficients are given below as;

2.8.2.1 Hard Thresholding

Hard thresholding is also called gating. It sets to zero all the coefficients of the function if they are less than the selected threshold value. For applying the hard thresholding to all the wavelets coefficient of the image for the elimination of noisy coefficients. Let T be the threshold value carried from noise variance. The Dohono defined the hard thresholding as [36],[25],[37],[38];

$$T_h = \begin{cases} x, & if\ |x| \geq \sigma \\ 0, & otherwise \end{cases} \qquad \text{............. (2.6)}$$

Here, x represents all the wavelet coefficients, higher than our selected threshold value and 0 represents noisy wavelet coefficients, which we are going to eliminate.

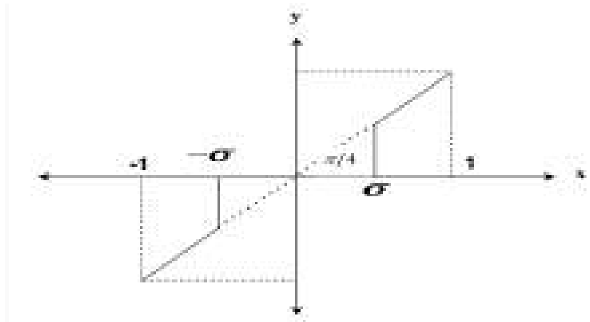

Fig.2.4: Hard Thresholding

2.8.2.2 Soft Thresholding

Mathematically Dohono defined the soft thresholding as[37],[38],[36];

$$T_s = \begin{cases} sig(x)(|x| - \sigma), & if \ |x| \geq \sigma \\ 0, & otherwise \end{cases} \qquad \text{............... (2.7)}$$

Here, sig is called the signum function

Soft thresholding also sets all the wavelet coefficients to zero, whose absolute value is lesser than the given threshold value T, then shrinking the non-zero wavelet coefficients toward zero. The threshold value T which we apply in the above thresholding techniques is called a universal threshold.

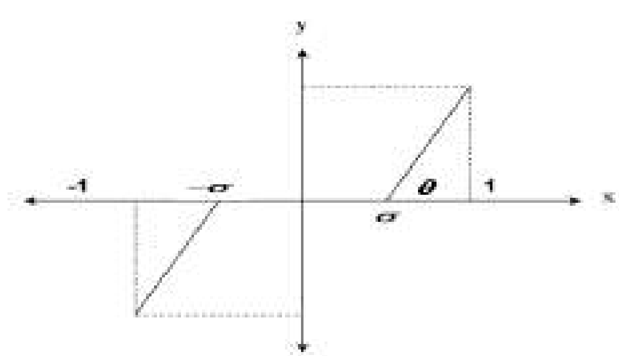

Fig.2.5: Soft Thresholding

Mathematically Dohono and John Stone define it as[38];

$$T = \sqrt{\sigma^2 \log_i N} \qquad \text{............... (2.8)}$$

Here σ^2 is called noise variance of detailed wavelet coefficients at the extreme level of decomposition. N is the size of the signal (i.e. a digital image may have a size of 256× 256) etc. The above T is sometimes called a Universal or VisuShrink threshold.

2.9 Proposed Algorithm for Digital Image Denoising

- **Step 1.** Load a grayscale (CT scan of chest) image in MATLAB (2020a software), with a size 255×255.
- **Step 2.** Add all four above-mentioned noises separately to the loaded image.
- **Step 3.** Apply all four filters separately to the images present in step 2.
- **Step4**. Apply wavelet transform ((DWT), sym4, coif2, db2 and bior1.5) to all the images present in step 3 at decomposition level 3.
- **Step 5.** Apply Thresholding (gbl) to the detailed coefficients of matrices present in step 4.
- **Step 6.** Apply Inverse Discrete Wavelet Transform (IDWT) to the matrices present in step 5. Finally, we get synthesized (reconstructed) images and we find out results between original and synthesized images on the bases of MSE, SNR, and PSNR.

The flow chart of the above algorithm is shown below;

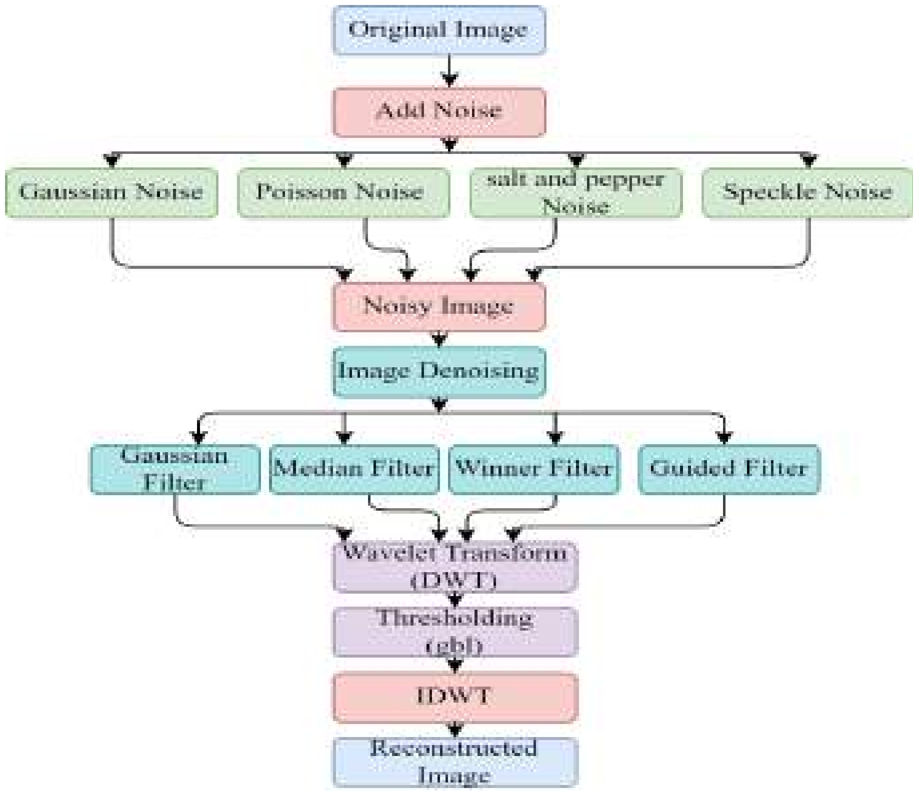

Fig.2.6: A flow Chart of the Algorithm

2.10 Table Analyses of MSE, SNR, and PSNR values for Image Denoising

Image	Gaussian filter with wavelet (sym4)			Median filter with wavelet (sym4)		
	MSE	SNR	PSNR	MSE	SNR	PSNR
Gaussian noise	222.7798	18.0714	24.6905	367.5120	15.8974	22.5120
Poisson noise	218.1478	18.1627	24.7773	368.4742	15.8861	22.5007
Salt & pepper noise	215.9369	18.2069	24.8215	366.2468	15.9006	22.5152
Speckle noise	216.5448	18.1947	24.8093	362.0227	15.9628	22.5774

Table no.2.1: Gaussian Filter with sym4, Median Filter with sym4

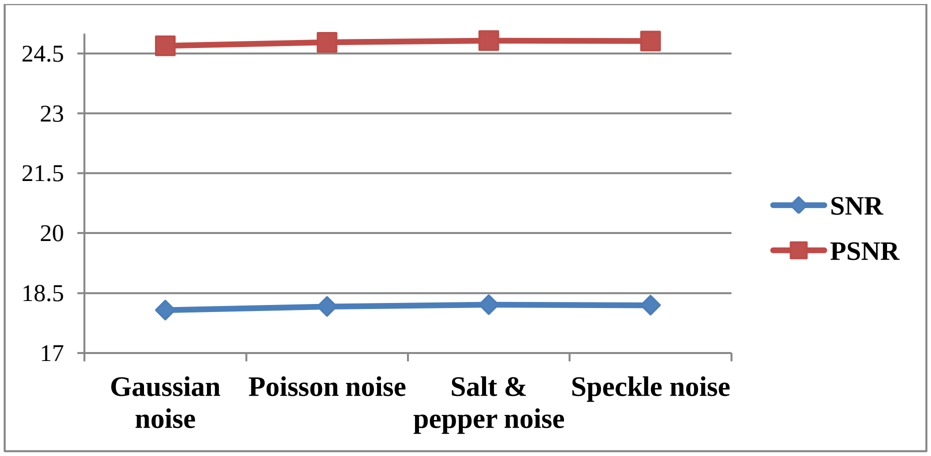

Fig.2.7: Gaussian Filter with sym4

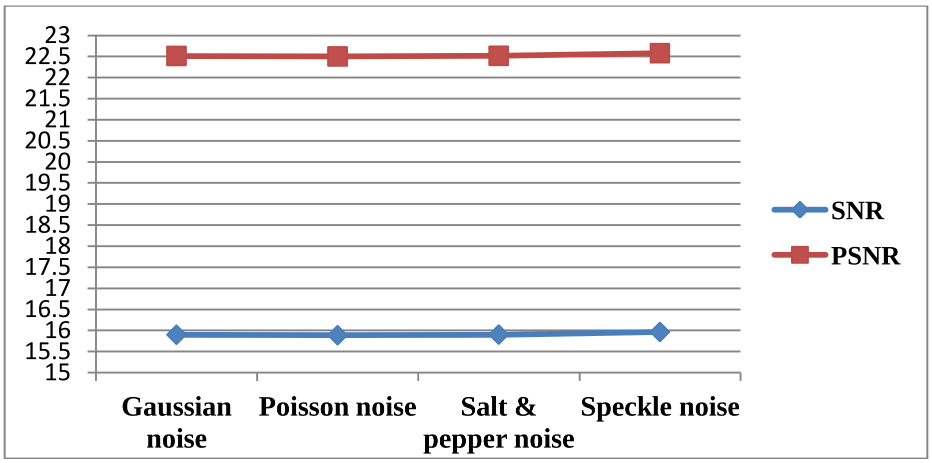

Fig.2.8: Median Filter with sym4

Image	Guided filter with Wavelet (sym4)			Winner filter with Wavelet (sym4)		
	MSE	SNR	PSNR	MSE	SNR	PSNR
Gaussian noise	166.777	9.5464	15.9444	63.7052	23.5084	30.1230
Poisson noise	165.457	9.3727	15.9873	57.2195	24.9747	30.8694
Salt & pepper noise	167.714	9.3031	15.9177	54.3127	24.2012	30.8158
Speckle noise	163.546	9.4066	16.0212	50.6327	24.5059	30.1205

Table no.2.2: Guided filter with sym4, Winner filter with sym4

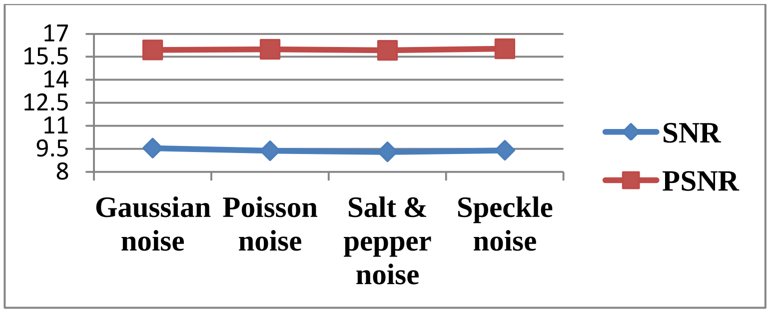

Fig.2.9: Guided Filter with sym4

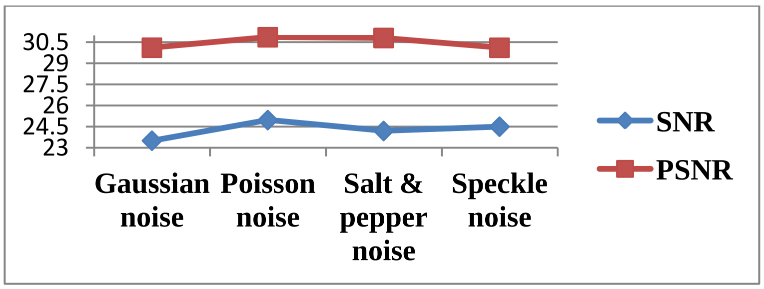

Fig.2.10: Winner Filter with sym4

Image	Gaussian filter with Wavelet (coif2)			Median filter with Wavelet (coif2)		
	MSE	SNR	PSNR	MSE	SNR	PSNR
Gaussian noise	233.6111	18.0553	24.6699	367.7112	15.8951	22.5097
Poisson noise	219.3934	18.1380	24.7526	367.014	15.9137	22.5283
Salt & pepper noise	217.9397	18.1669	24.7814	357.6676	16.0154	22.6300
Speckle noise	218.0626	18.7790	24.7790	361.2324	15.9723	22.5869

Table no.2.3: Gaussian Filter with coif2 & Median filter with coif2

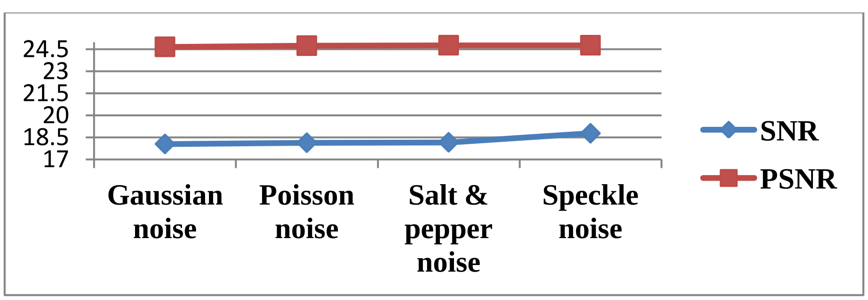

Fig.2.11: Gaussian Filter with coif2

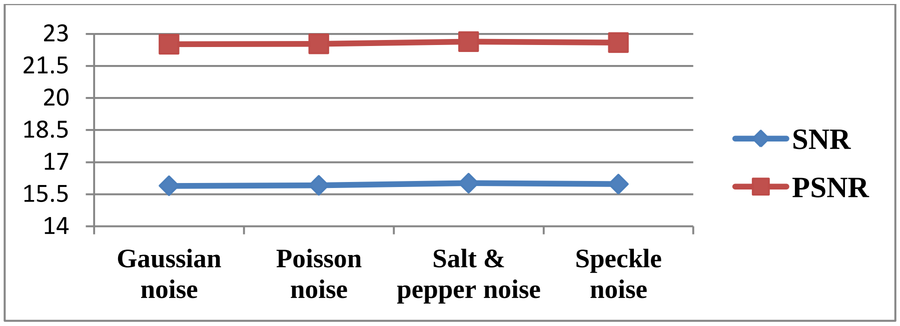

Fig.2.12: Median Filter with coif2

Image	Guided filter with Wavelet (coif2)			Winner filter with Wavelet (coif2)		
	MSE	SNR	PSNR	MSE	SNR	PSNR
Gaussian Noise	166.565	9.3321	15.9468	63.4031	23.5291	30.1437
Poisson Noise	165.456	9.3789	15.9936	70.9913	24.6166	30.8312
Salt & Pepper Noise	167.425	9.3003	15.9149	52.8049	24.3234	30.7381
Speckle Noise	164.102	9.3991	16.0137	50.5920	24.5094	30.1240

Table no.2.4: Guided filter with coif2 & Winner filter with coif2

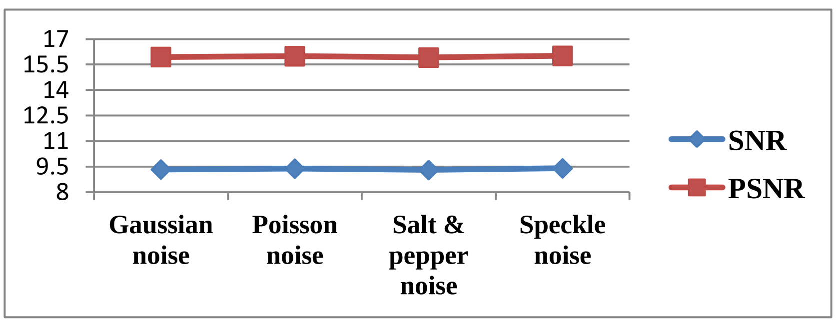

Fig.2.13: Guided Filter with coif2

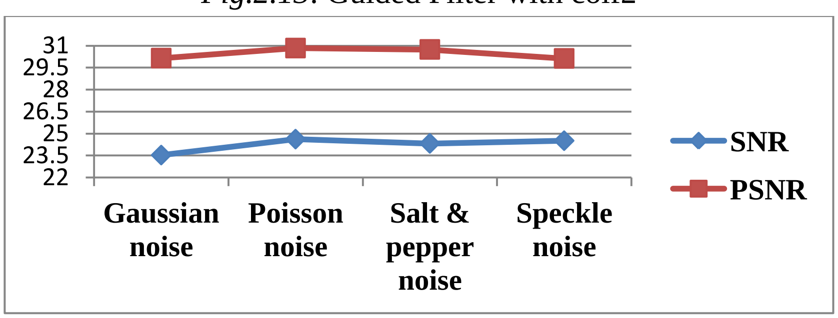

Fig.2.14: Winner Filter with coif2

Image	Gaussian filter with Wavelet (db2)			Median filter with Wavelet (db2)		
	MSE	SNR	PSNR	MSE	SNR	PSNR
Gaussian noise	227.9909	17.9710	24.5856	368.145	15.8835	22.4981
Poisson noise	223.4957	18.0575	24.6721	367.7662	15.8945	22.5091
Salt & pepper noise	221.8690	18.0892	24.7038	362.2767	15.9598	22.5744
Speckle noise	221.7060	18.7069	24.7070	346.457	15.9941	22.8614

Table no.2.5: Gaussian filter with db2 & Median filter with db2

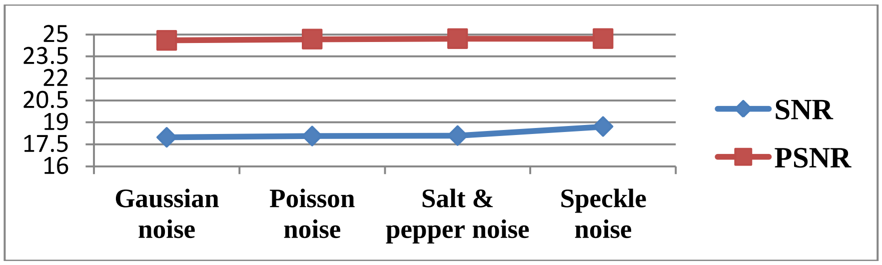

Fig.2.15: Gaussian Filter with db2

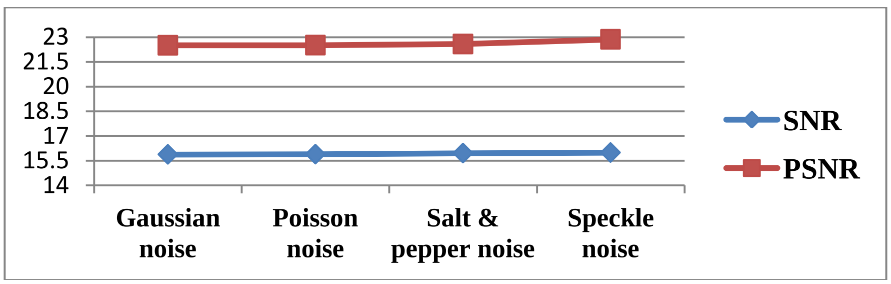

Fig.2.16: Median Filter with db2

Image	Guided filter with Wavelet (db2)			Winner filter with Wavelet (db2)		
	MSE	*SNR*	*PSNR*	MSE	SNR	PSNR
Gaussian noise	166.600	9.3335	15.9481	65.3762	23.3960	30.0106
Poisson noise	165.455	9.3644	15.9753	60.9272	24.5470	30.4616
Salt & pepper noise	166.154	9.2966	15.9112	51.9088	24.3978	30.0124
Speckle noise	164.125	9.4041	16.0187	54.4555	24.3956	30.0102

Table no.2.6: Guided filter with db2 & Winner filter with db2

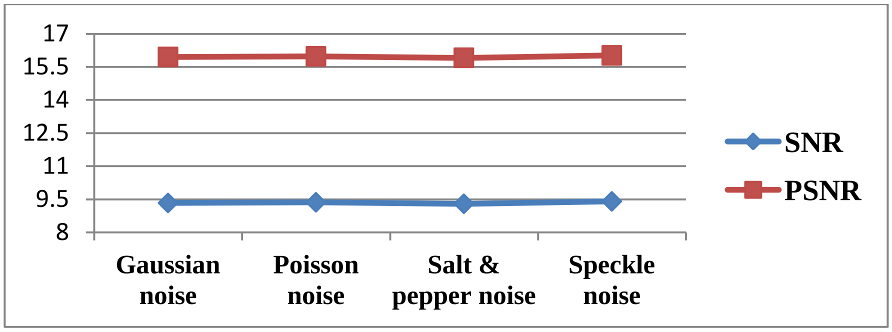

Fig.2.17: Guided filter with db2

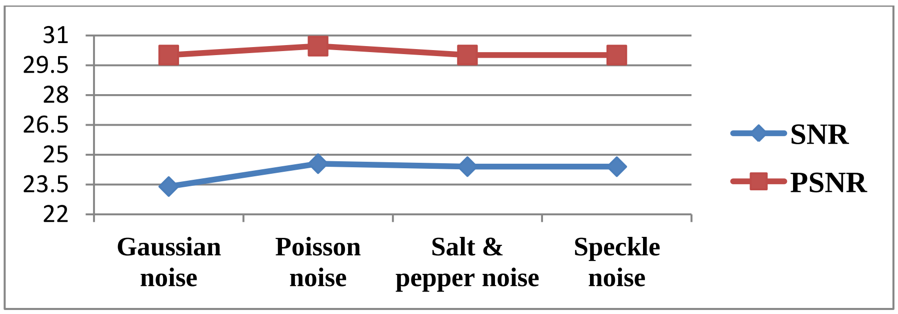

Fig.2.18: Winner filter with db2

Image	Gaussian filter with Wavelet (bior1.5)			Median filter with Wavelet (bior1.5)		
	MSE	SNR	PSNR	MSE	SNR	PSNR
Gaussian noise	223.3576	18.0602	24.6748	368.6849	15.8830	22.4982
Poisson noise	218.9749	18.1463	24.7609	364.9486	15.9279	22.5425
Salt & pepper noise	216.6532	18.1925	24.8071	357.9440	16.0120	22.6266
Speckle noise	216.8145	18.1893	24.8039	363.0442	15.9506	22.5652

Table no.2.7: Gaussian filter with bior1.5 & Median filter with bior1.5

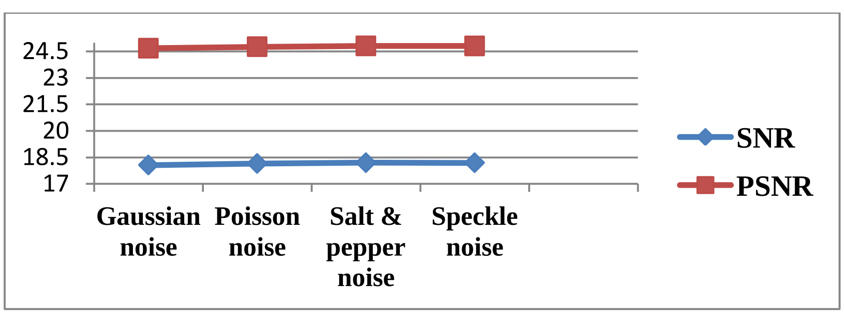

Fig.2.19: Gaussian filter with bior1.5

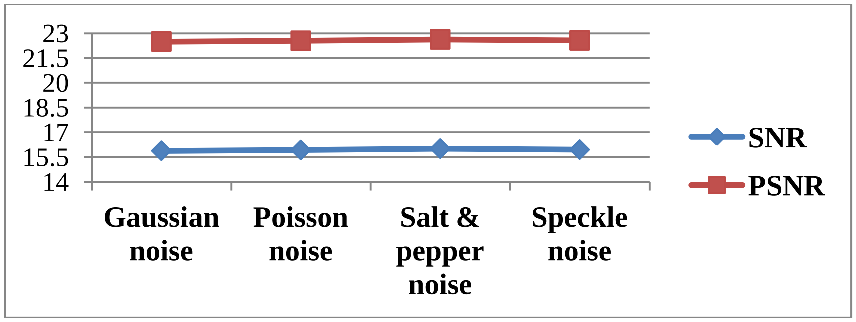

Fig.2.20: Median filter with bior1.5

Image	Guided filter with Wavelet (bior1.5)			Winner filter with Wavelet (bior1.5)		
	MSE	SNR	PSNR	MSE	SNR	PSNR
Gaussian noise	166.456	9.3417	15.9563	64.9217	23.4263	30.0409
Poisson noise	164.215	9.3659	15.9805	59.2671	24.8222	30.8368
Salt & pepper noise	164.524	9.3075	15.9221	52.2470	24.3696	30.6842
Speckle noise	161.456	9.3984	16.0130	59.2972	24.4493	31.0639

Table no.2.8: Guided filter with bior1.5 & Winner filter with bior1.5

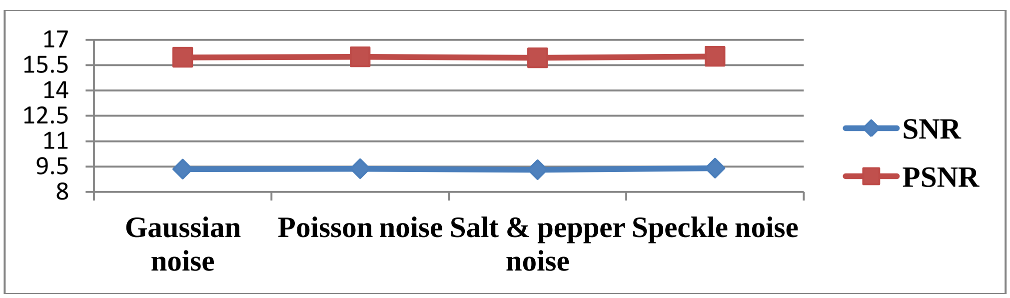

Fig.2.21: Guided filter with bior1.5

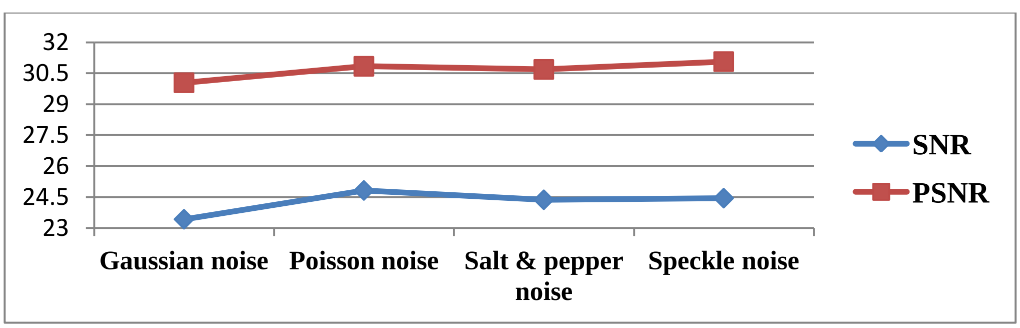

Fig.2.22: Winner filter with bior1.5

2.11 Test Image (Input image) and Synthesized (Output image) Image

The CT scan of the chest is selected as a test image, as it is used in different research articles[34],[39],[40],[41].

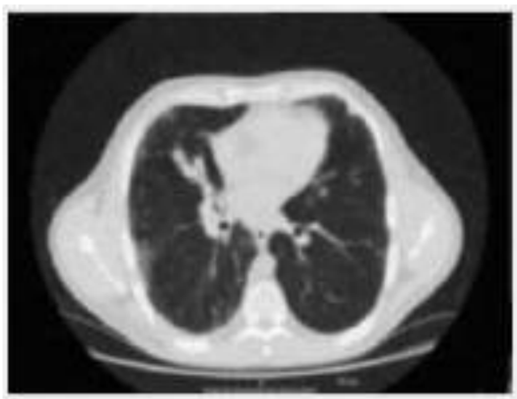

Fig.2.23: Noisy Test Image (Poisson Noise)

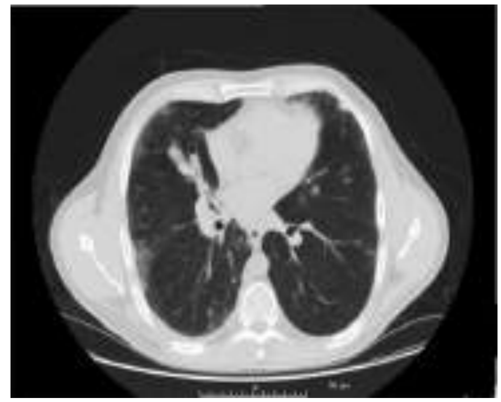

Fig.2.24: Image Winner filter and Wavelet (sym4)

2.12 Summary

The whole simulation (work) is done through MATLAB (2020a) software. In the given algorithm we use four simulated noises, four digital image denoising filters, and four discrete wavelets for CT-scan image denoising. A comparative study was done on image denoising performance bases. The image denoising performance is based on three parameters, especially on SNR and PSNR i.e. “Signal Noise Ratio, and Peak Signal to Noise Ratio”. We say higher is the SNR higher is denoising performance between the input and denoised image. Each filter with wavelets performs better in terms of CT scan image denoising. But from table2.2, table2.4, table2.6, and table2.8, the wavelets, i.e. sym4, coif2, db2, and bior1.5 with winner filter perform better in terms of image denoising on passion simulated noise on the bases SNR values than the rest wavelets.

Chapter 3

DIGITAL IMAGE DENOISING BY A THRESHOLDING TECHNIQUE AND WAVELET PACKET TRANSFORMATION

3.1 Introduction

Noise entrance in digital images during capturing, transmission, and storing is a major concern[42]. Different techniques and methods have been applied to remove the noise from noisy images[8],[43].The wavelet transform is a well-known method for analyzing both stationary and non-stationary signals[44]. It represents the signal in the frequency domain as well as in the time domain[45]. The key concept behind the wavelet transform is that it defines the analyzing signal into 2-categories of coefficients[36]. The higher size coefficients denote original data, and smaller coefficients denote noise present in it. These coefficients are generated because the wavelet decomposition is a procedure of translation and scaling of a given signal. Scaling means stretching and squeezing the window function (or wavelet) to detect the sharp spikes, discontinuities and translation means shifting of window function across the signal on the entire time axis[45].

In this chapter, we denoised a digital image of a cameraman of size 256×256[46]. First, we applied wavelet transform to decompose the given function (Image) up to three levels, here we get a threshold value from detailed coefficients, and then finally we decompose the same function by wavelet packet transform and applied a threshold value obtained from wavelet transformation. In other words, we applied three parameters wavelet transform, thresholding, and wavelet packet transform, to remove noise from the given digital image.

We have already discussed wavelets and wavelet transform in first chapter. Here we add that wavelets have the potential to cut down a large signal into small pieces or wavelets, and each piece is then analyzed with a resolution that matches its scale[44],[47],[48],[49].

Wavelets are applied for different purposes, like signal compression, smoothing, sharpening, denoising, etc. It is important to denoise almost every type of signal. A wavelet has its shape, identity, and varied characteristics. There is no particular rule for the selection of wavelets for any signal. Wavelet performance in terms of signal analyses varied from signal to signal i.e., all wavelets cannot perform better at every time for any signal.According to the different latest research articles and research trends we have selected a digital image of a cameraman as a test image which is a non-stationary signal. Wavelet transform decomposes the input signal into four types of coefficients at a level first i.e. "approximation coefficient, horizontal coefficients, vertical coefficients, and diagonal coefficients". But at a level second, only the approximation part of the signal is further decomposed. The same process is applied at the next levels until we apply the inverse wavelet transform[44],[49],[50].

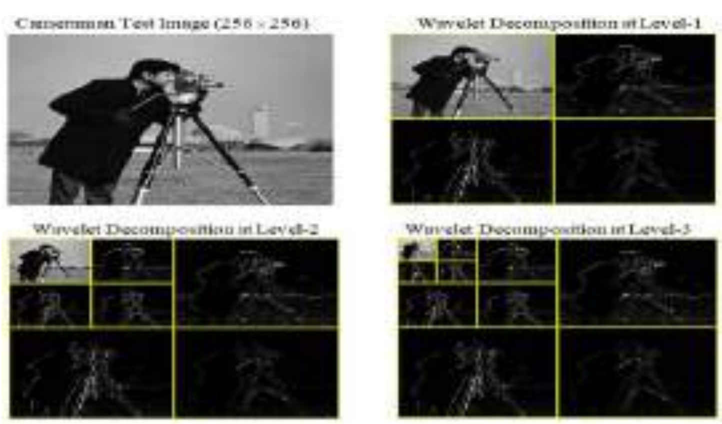

Fig.3.1: DWT of the Cameraman Image up to level-3

The different decomposition levels of an input image by DWT are given above;

In the WT process, there is a chance of loss of some information from the same input signal. But in the case of wavelet packet transform (WPT) which is a generalization of the wavelet transform, where all the coefficients i.e. “approximation coefficient, horizontal coefficients, vertical coefficients, and diagonal coefficients” are further decomposed at each level[47].The tree decomposition of a signal by WPT is given as;

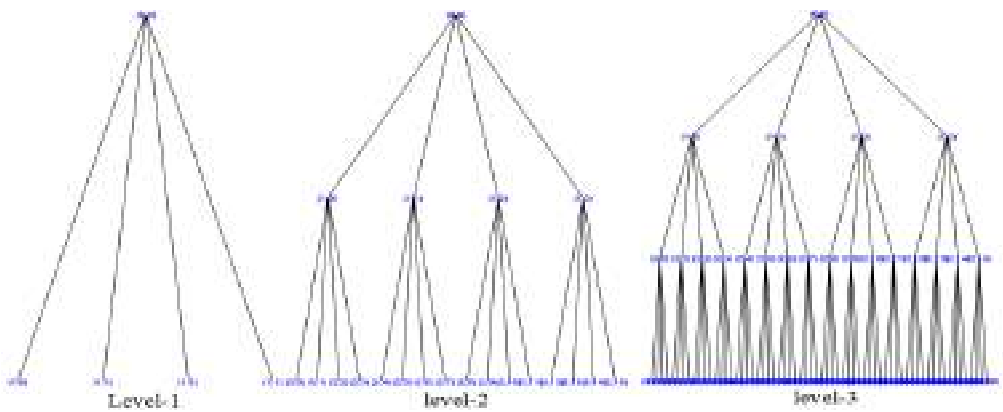

Fig.3.2: WPD at level-1, level-2, and level-3

The difference between wavelet transform and wavelet packet transform is shown below[44],[50],[36];

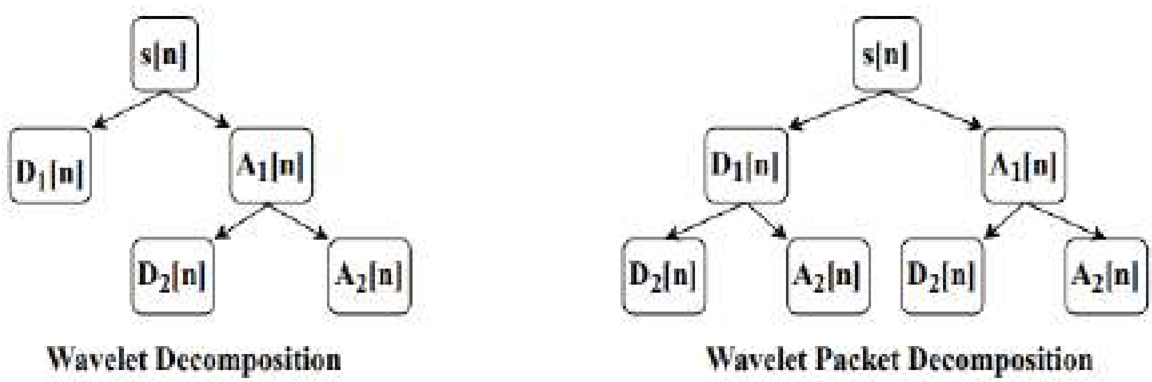

Fig.3.3: Wavelet and Wavelet Packet Decomposition of the Signal s[n]

Here s[n] is original signal, A[n] is approximation coefficient and D[n] is detailed coefficients.

In the same chapter, we have used both wavelet transform and wavelet packet transform separately and finally use the IWPT to reconstruct the synthesized image for the denoising purpose.

3.2 Proposed Thresholding Technique

We have already defined a thresholding technique for noise removal in the second chapter of this thesis. Several types of thresholding techniques like Soft, Hard, Hybrid, SureShrink, and VisuShrink BayeShrink are utilized to denoise the digital images[51][16][38]. The general thresholding function is defined as[52];

$$\lambda(n) = \sigma\sqrt{n\log(M)} = T \quad , n \, \varepsilon \, N \qquad \text{………….} \ (3.1)$$

Here M is the size of the image, like 128×128, 240×240, 256×256 etc, σ is the standard deviation of the detailed coefficients at finer levels. “It is an increasing function of ‘n’ ”, let us put n=1, in function (3.1) we “get

$$\lambda_1 = \sigma\sqrt{\log(M)} \qquad \text{………………...} \ (3.2)$$

M is the image size, σ^2 is variance obtained from detailed coefficients, λ or T is called the threshold value

Put n = 2, in function (3.1), we get

$$\lambda_2 = \sigma\sqrt{2\log(M)}$$

Here λ or T is called as Universal or SureShrink threshold[53][36][52]”.

Now, Put= 2, we get

$\lambda_3 = \sigma\sqrt{3\log(M)}$ ………………..

(3.3)

Now applied the above three threshold function λ_1, λ_2 *and* λ_3 separately and took the average threshold value i.e. $\lambda_1 + \lambda_2 + \lambda_3 / 3$.

3.3 Proposed method

The whole is work done through MATLAB (2020a) software. The proposed method consists of the following steps,

1. Load an image of the cameraman with size 256×256 and converted it to a grayscale image.
2. The input image is decomposed by wavelet transform up to level 3.
3. At decomposition level 3 of a given input image, determine the threshold from the detailed coefficients by the above-given formula and their average for comparison purposes in the next step.
4. Now decompose the original image of the cameraman by WPT and apply the threshold value obtained in step 3.
5. Finally, applying IWPT for the reconstruction of the synthesized image, and comparing the results of MSE, SNR, and PSNR between input and synthesized images. Here

$$MSE = \frac{1}{M \times N} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[f(x,y) - \hat{f}(x,y) \right]^2$$

$$SNR = 20\log_{10} \frac{\sum_{x=0}^{m-1} \sum_{y=0}^{n-1} [\hat{f}(x,y)]^2}{\sum_{x=0}^{m-1} \sum_{y=0}^{n-1} [f(x,y) - \hat{f}(x,y)]^2}$$

$$PSNR = 10\log_{10} \frac{(256)^2}{MSE}$$

Here $f(x,y)$ is an input image, $f(x,y)$ is output or synthesized image, 256×256 is a size of the $f(x,y)$ [36].

The process of the proposed method is shown below;

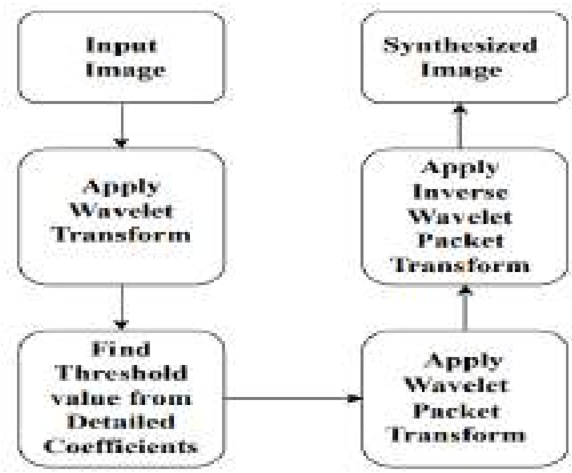

Fig.3.4: Flow chart of the above-proposed method.

3.4 Table Data Analyses in terms of SNR and PSNR

Size of the cameraman Image (256×256)		Wavelet Transform (Haar)		
Threshold value	λ_1	λ_2	λ_3	Average Threshold
σ = 0.227	0.499	1.549	1.897	1.315

Table no.3.1: Average threshold value obtained by WT (haar) from detailed coefficients

Wavelet Packet Transform (haar) at $\lambda = 1.315$		
MSE	Between the Input and Synthesized Image of the Cameraman	90.9403
SNR		18.1690
PSNR		28.5895

Table no.3.2: Wavelet Packet Transform (haar) at $\lambda = 1.315$

Size of the cameraman Image (256×256)		Wavelet Transform (sym4)		
Threshold value	λ_1	λ_2	λ_3	Average Threshold
σ = 0.2604	0.571	1.773	2.172	1.505

Table no.3.3: Average threshold value obtained by WT (sym4) from detailed coefficients

Wavelet Packet Transform (sym4) at $\lambda = 1.505$		
MSE	Between the Input and Synthesized Image of the Cameraman	90.3907
SNR		18.2440
PSNR		28.6759

Table no.3.4: Wavelet Packet Transform (sym4) at $\lambda = 1.505$

Size of the cameraman Image (256×256)		Wavelet Transform (sym6)		
Threshold value	λ_1	λ_2	λ_3	Average Threshold
σ = 0.2463	0.540	1.677	2.054	1.423

Table no.3.5: Average threshold value obtained by WT (sym6) from detailed coefficients

Wavelet Packet Transform(sym6) at $\lambda = 1.423$		
MSE	Between the Input and Synthesized Image of the Cameraman	90.8933
SNR		18.2161
PSNR		28.6036

Table no.3.6: Wavelet Packet Transform (sym6) at $\lambda = 1.423$

Size of the cameraman Image (256×256)		Wavelet Transform (sym8)		
Threshold value	λ_1	λ_2	λ_3	Average Threshold
σ = 0.2371	0.520	1.615	1.977	1.370

Table no.3.7: Average threshold value obtained by WT (stm8) from detailed coefficients

Wavelet Packet Transform(sym7) at $\lambda = 1.370$		
MSE	Between the Input and Synthesized Image of the Cameraman	91.1485
SNR		18.2047
PSNR		28.5679

Table no.3.8: Wavelet Packet Transform (sym8) at $\lambda = 1.370$

Size of the cameraman Image (256×256)		Wavelet Transform (db4)		
Threshold value	λ_1	λ_2	λ_3	Average Threshold
σ = 2604	0.571	1.733	2.172	1.505

Table no.3.9: Average threshold value obtained by WT (db4) from detailed coefficients

Wavelet Packet Transform(db4) at $\lambda = 1.505$		
MSE	Between the Input and Synthesized Image of the Cameraman	90.6363
SNR		18.2357
PSNR		28.5098

Table no.3.10: Wavelet Packet Transform (db4) at $\lambda = 1.505$

Size of the cameraman Image (256×256)		Wavelet Transform (coif2)		
Threshold value	λ_1	λ_2	λ_3	Average Threshold
σ = 0.2463	0.540	1.677	2.054	1.423

Table no.3.11:Average threshold value obtained by WT (coif2) from detailed coefficients

Wavelet Packet Transform(coif2) at $\lambda = 1.423$		
MSE	Between the Input and Synthesized Image of the Cameraman	90.8420
SNR		18.1911
PSNR		28.5888

Table no.3.12: Wavelet Packet Transform (coif2) at $\lambda = 1.423$

Size of the cameraman Image (256×256)		Wavelet Transform (coif5)		
Threshold value	λ_1	λ_2	λ_3	Average Threshold
σ = 0.1869	0.410	1.273	1.559	1.081

Table no.3.13: Average threshold value obtained by WT (coif5) from detail coefficients

Wavelet Packet Transform(coif5) at $\lambda = 1.081$		
MSE	Between the Input and Synthesized Image of the Cameraman	93.2874
SNR		18.0836
PSNR		28.4653

Table no.3.14: Wavelet Packet Transform (coif5) at $\lambda = 1.081$

Size of the cameraman Image (256×256)		Wavelet Transform (bior2.4)		
Threshold value	λ_1	λ_2	λ_3	Average Threshold
σ = 0.2561	0.562	1.744	2.136	1.480

Table no.3.15: Average threshold value obtained by WT (bior2.4) from detail coefficients

Wavelet Packet Transform(bior2.4) $\lambda = 1.480$		
MSE	Between the Input and Synthesized Image of the Cameraman	89.6341
SNR		18.2308
PSNR		28.6617

Table no.3.16: Wavelet Packet Transform (bior2.4) at $\lambda = 1.480$

Size of the cameraman Image (256×256)		Wavelet Transform (bior2.6)		
Threshold value	λ_1	λ_2	λ_3	Average Threshold
σ = 0.2371	0.520	1.615	1.977	1.370

Table no.3.17: Average threshold value obtained by WT (bior2.6) from detailed coefficients

Wavelet Packet Transform(bior2.6) $\lambda = 1.370$		
MSE	Between the Input and Synthesized Image of the Cameraman	90.8283
SNR		18.2237
PSNR		28.6197

Table no.3.18: Wavelet Packet Transform (bior2.6) at $\lambda = 1.370$

Size of the cameraman Image (256×256)		Wavelet Transform (bior2.8)		
Threshold value	λ_1	λ_2	λ_3	Average Threshold
σ = 0.2250	0.493	1.532	1.877	1.301

Table no.3.19: Average threshold value obtained by WT (bior2.8) from detailed coefficients

Wavelet Packet Transform(bior2.8) $\lambda = 1.301$		
MSE	Between the Input and Synthesized Image of the Cameraman	90.0804
SNR		18.1869
PSNR		28.6725

Table no.3.20: Wavelet Packet Transform (bior2.8) at $\lambda = 1.301$

Size of the cameraman Image (256×256)		Wavelet Transform (bior3.3)		
Threshold value	λ_1	λ_2	λ_3	Average Threshold
σ = 0.2604	0.571	1.773	2.172	1.505

Table no.3.21: Average threshold value obtained by WT (bior3.3) from detailed coefficients

Wavelet Packet Transform(bior3.3) $\lambda = 1.505$		
MSE	Between the Input and Synthesized Image of the Cameraman	88.0702
SNR		18.23801
PSNR		28.8607

Table no.3.22: Wavelet Packet Transform (bior3.3) at $\lambda = 1.505$

MSE, SNR, and PSNR values between the input image and synthesized image at an average threshold value (λ) through WPT						
s. no	Wavelets		λ	MSE	SNR	PSNR
01.	haar	Wavelet Packet Transform	1.315	90.9403	18.1690	28.5895
02.	sym4		1.505	90.3907	**18.2440**	**28.6759**
03.	sym6		1.423	90.8433	18.2116	28.6036
04.	sym8		1.370	91.1485	18.2047	28.5679
05.	db4		1.504	90.6363	**18.2357**	28.5098
06.	coif2		1.423	90.8420	18.1911	28.5888
07.	coif5		1.081	93.2874	18.0836	28.4653
08.	bior2.4		1.480	89.6341	18.2308	28.6617
09.	bior2.6		1.370	90.8283	18.2237	28.6197
10.	bior2.8		1.301	90.0804	18.1869	**28.6725**
11.	bior3.3		1.506	88.0702	**18.2780**	**28.8607**

Table no.3.23: MSE, SNR, and PSNR values between the Input and Synthesized Image through Wavelet Packet Transformation

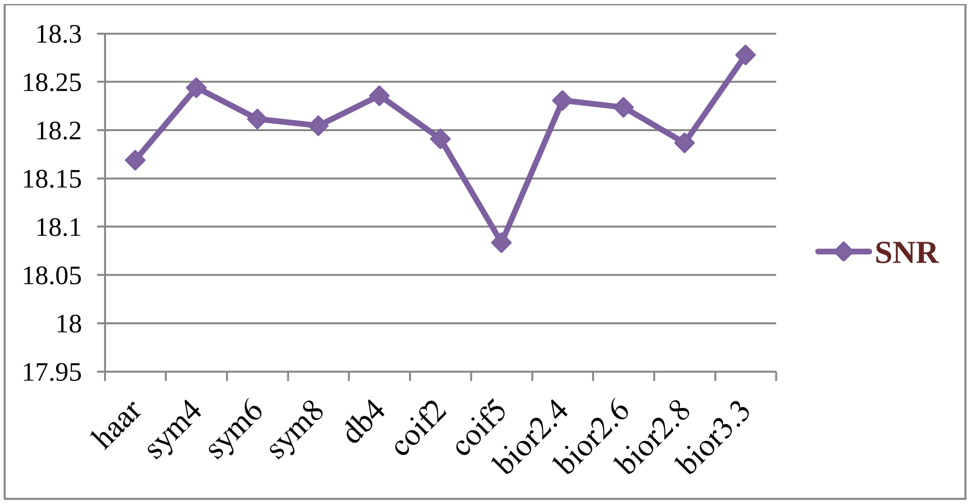

Fig.3.5: Graph Comparison of SNR values of the Eight Wavelets

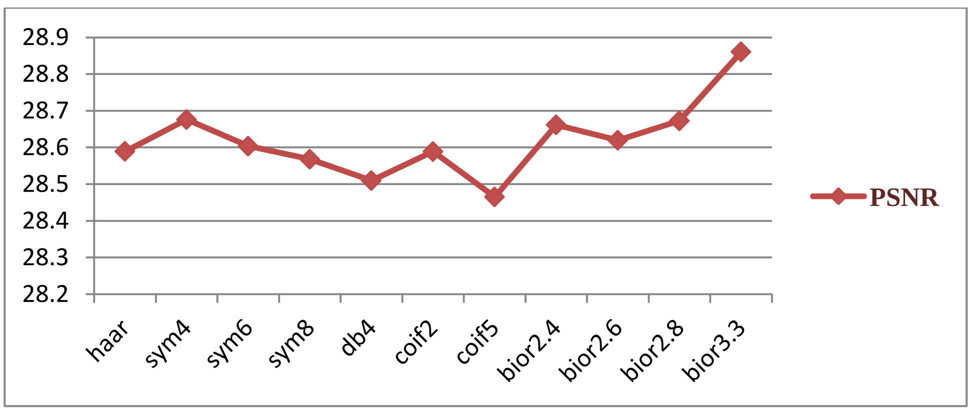

Fig.3.6: Graph Comparison of PSNR values of the Eight Wavelets

3.5 Original Image and Synthesized Image

We choose an image of a cameraman of size 256×256 as our test image because it is used as a test image in several research articles[46],[52],[54],[55],[56],[57],[58],[59]. We have denoised it and obtained a synthesized image by applying the above-proposed method.

Fig.3.7: Noisy and Denoised Image of Cameraman

3.6 Summary

Here we have proposed an algorithm to denoise a digital image. For digital image denoising purposes, we have selected an image of a cameraman of size 256×256 as our test image, which are used in several research articles. Here we observe, higher the threshold value higher the SNR and PSNR. We know higher the SNR and PSNR better is signal denoising. Finally a performance comparison of wavelets in terms of digital image denoising is done. According to the Table no.2.23 and Fig.3.6 the wavelet bior3.3 performs better at rank-1,sym4 at rank-2,db4 at rank-3,bior2.4 at rank-4,bior2.6 at rank-5,sym6 at rank-6,sym8 at rank-7,coif2 at rank-8,bior2.8 at rank-9, haar at rank-10 and coif5 at rank-11 in terms of SNR values. On other side according to the Table no.2.23 and Fig.3.7 the wavelet bior3.3 performs better at rank-1, sym4 at rank-2, bior2.8 at rank-3, bior2.4 at rank-4,bior2.6 at rank-5,sym6 at rank-6,haar at rank-7,coif2 at rank-8,sym8 at rank-9, db4 at rank-10 and coif5 at rank-11 in terms of PSNR values.

Chapter 4

ESTIMATED THRESHOLD FOR DIGITAL IMAGE DENOISING THROUGH WAVELET PACKET TRANSFORMATION

4.1 Introduction

Thresholding is a popular technique applied for digital image denoising. Different types of signals have been improved by the thresholding technique in terms of enhancement, compression, and denoising [60],[61],[62],[63],[64],[65]. The thresholding technique basically divides the input function (image) into two types of coefficients: High-value coefficients and Lower value coefficients[66]. Then, it may consider the small coefficients of the signal are noise (unwanted data) present in it and the rest coefficients as the real or original signal. Thresholding is basically a signal-improving technique[65]. There are several ways to select the threshold, depending on the method and process. A good threshold selection yields better results i.e., once we select a perfect threshold, better results will be produced in terms of signal denoising. In other words, the optimal threshold value always performs better in terms of digital image smoothing, edge preservation, compression, and denoising. Therefore, in digital image processing, the thresholding technique depends on how the better optimal threshold value or threshold function is be obtained[67].

Wavelet thresholding or shrinkage is considered to be an advanced technique for digital image denoising. If we have selected a method for thresholding and have an optimal threshold or threshold function, we can apply it in the following manner.

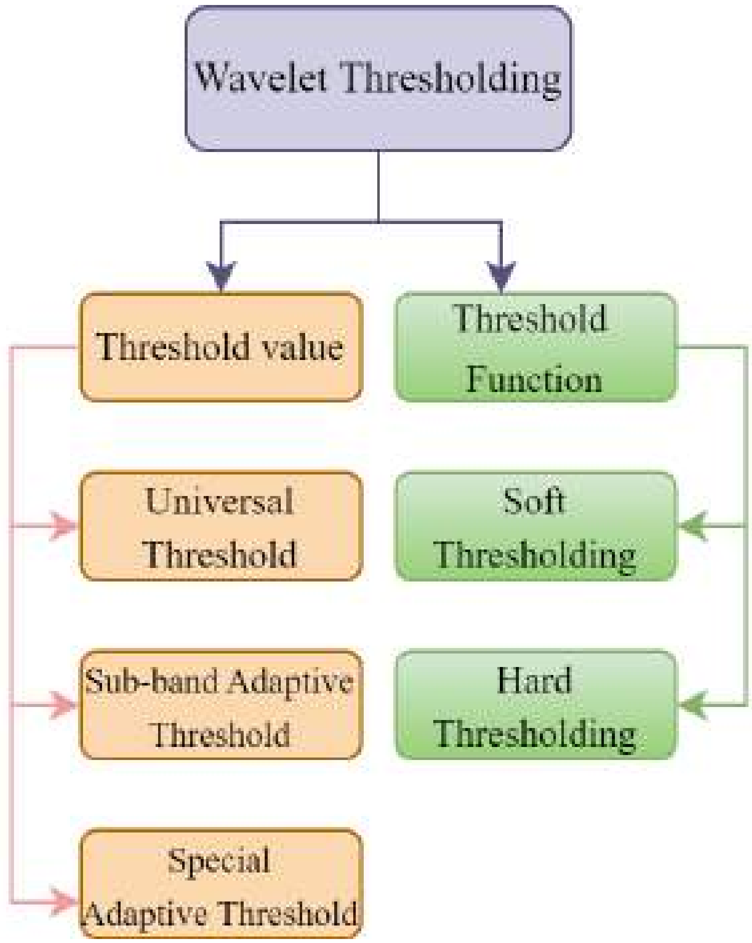

Fig.4.1: Different ways of Thresholding

As already mentioned in the chapter second thresholding function are of two types i.e**.** **"**hard thresholding and soft thresholding technique[66][65]. In HT all coefficients are put to 0 if they are less than the fixed threshold (T), and the rest coefficients are taken unchanged[53]. It is defined as

$$W_p(t) = \begin{cases} W_q(t), \; for \; |W_q(x)| \geq T \\ 0 \quad , \; otherwise \end{cases} \qquad \text{…………..} \; (4.1)$$

Here, T is a fixed threshold. The two functions $W_p(t)$ and $W_q(t)$ denotes de-noised and noisy wavelet coefficients. But in soft-thresholding the coefficients less than threshold value T are set to be zero. The rest essential coefficients are reduced by absolute threshold value and are shown as;

$$W_p(t) = \begin{cases} \mathrm{sgn}(W_q(t))(W_q(t) - T, & for\left|W_q(t)\right| \geq T \\ 0 & , otherwise \end{cases} \quad \text{………….. (4.2)}$$

Here, sgn is called the signum function, it goes back to one, if the element is bigger than zero, zero if it is equal to zero, and -1 if it is less than zero. Also, the function $W_p(t)$ $W_p(t)$ and T are already defined in the above function "[16]. Noise present in the signals can be in additive or multiplicative form[35],[67] i.e.

$$\alpha(x, y) = \beta(x, y) + \eta(x, y) \quad \text{…………….. (4.3)}$$

$$\alpha(x, y) = \beta(x, y) \times \eta(x, y) \quad \text{…………….. (4.4)}$$

Here, "(x, y)denotes the image pixel position, $\eta(x, y)$ which is an unwanted signal (noise) executed (add or multiple) with the original image $\beta(x, y)$ and created a noisy image $\alpha(x, y)$. We have already mentioned in chapter second that various noises entered in the digital images due to certain reasons, and affect the digital image. Therefore different methods were adopted to de-noise such noisy images".

4.2 Noise Shrinkage Functions

In digital image processing, different types of noise shrinkage (reduction) functions are available. Some of the important functions are mentioned below;

4.2.1 SureShrink

SureShrink is sometimes called a hybrid of the "Universal threshold[65],[68],[69],[70]. This thresholding technique is based on the principle of Stein's Unbiased Risk Estimator (SURE). In other words, while applying soft thresholding it is used to "minimize the stein's unbiased risk estimate". Sometimes it is applied to minimize mean square error (MSE). The threshold value of t_j at each resolution level j of the said used wavelet transform, it is opted to level-dependent thresholding. The threshold value of SureShrink (t_S) is defined as[65]";

$$t_s = \min(t, \hat{\sigma}_n \sqrt{2\log n}) \quad \text{……………..} (4.5)$$

The value of **t** in (4.5) is used to minimize "Stein's Unbiased Risk Estimator (SURE)''.

4.2.2 Bayes Shrink

In digital image processing, BayesShrink is a versatile method for image de-noising[69]. It gives thrust on "the minimization Bayesian risk, so it is called BayesShrink. As we know that the noise is Additive in Nature". Therefore, a polluted signal is an "Additive Sum of Original Image and Noisy one". It can be shown in terms of variances,

$$\sigma_{\alpha_1}{}^2 = \sigma_{\alpha_2}{}^2 + \sigma_{\alpha_3}{}^2 \quad \text{……………..} (4.6)$$

Where $\sigma_{\alpha_1}{}^2$ and $\sigma_{\alpha_2}{}^2$ denotes variance of the damaged and original image, while as $\sigma_{\alpha_3}{}^2$ denotes variance of the noise. Therefore the estimated threshold function or Bayesian threshold is defined as

$$t_B = \frac{\sigma_{\alpha_3}{}^2}{\sigma_{\alpha_2}{}^2} \quad \text{……………..} (4.7)$$

Here, ${\sigma_{\alpha_2}}^2$ is calculated as

$${\sigma_{\alpha_2}}^2 = \sqrt{\max({\sigma_{\alpha_1}}^2 - {\sigma_{\alpha_3}}^2, 0)} \qquad \text{…………….. (4.8)}$$

By BayesShrink, we mean to get a better outcome (Image) as well as to de-noise the smooth region of the image by applying thresholding ("at each sub-band of the wavelet decomposition").

4.2.3 VisuShrink

The VisuShrink is another broadly applied method for digital image denoising[69]. "The value of VisuShrink t_V is corresponding to the "standard deviation of the noise". The function of the VisuShrink method is defined as;

$$t_V = \sigma\sqrt{2\log n} \qquad \text{…………….. (4.9)}$$

Here, n is the size of the signal, "sigma called the standard deviation of the noise, and is calculated as[69],[70];

$$\sigma = \frac{Median\left\{\left|w_{i=1,k}\right| : k = 0,1,.....,2^{i-1}\right\}}{0.6745} \qquad \text{…………….. (4.10)}$$

Here, the coefficient $w_{i=1,k}$ is corresponding to, HL, LH, and HH ("Wavelet Detail Coefficients)". "The VisuShrink is especially well identified for providing very soft improved signals. That is why, because it removes excessive coefficients"[9],[65].

4.2.4 New thresholding function

Here we introduce a new thresholding technique based on global thresholding for digital image denoising. The threshold value (or sigma value) is obtained from detail coefficients (LH, HL, HH) of the image when wavelet transform is applied for decomposition of the image at finer level-3. The given Thresholding function is defined below as;

$$\lambda(n) = \sigma\sqrt{n\log M} \Big/ 3^{n/3} \;\; ; n \in N \qquad \text{…………….. (4.11)}$$

Where n is the number of decomposition levels, M is the size of the original or denoised image, and the value of σ (standard deviation) is calculated by the above function (4.10).

4.3 Proposed Algorithm

The whole simulation (work) is done through MATLAB (2020a) software under these steps.

Step1. A grayscale image of Lena of size (256×256) is taken as a test image.

Step2. Adding (Simulated) Gaussian Noise (0.01) of the above input image.

Step3. Then this digital noisy image is broken down (decomposed) into approximation and detailed coefficients by various wavelets at level 3.

Step4. Find out threshold value from detailed coefficient at standard deviation (**σ**) by threshold function.

Step5. For de-noising the noisy digital image, we apply WPT with the threshold obtained from step 4.

Step6. Now apply IWPT to reconstruct the synthesized (de-noised) image for analysis purposes.

Step7. Finally, compute the results of SNR and PSNR to measure the de-noising ratio between the original and synthesized image.

Here, MSE, SNR, and PSNR are calculated by the below formulas[65],[69];

Here 256×256 is the size of the input image, it has 0 to 256 gray shades, $f(x,y)$ is an input noisy image, and $f(x,y)$ is a reconstructed (de-noised) image.

Here MSE, SNR and PSNR are calculated as, MSE= $\frac{1}{M\times N}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f(x,y)-\hat{f}(x,y)]^2$,

PSNR= $10\times\log_{10}\frac{255^2}{MSE}$ and SNR= $\frac{1}{M\times N}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f(x,y)-\hat{f}(x,y)]^2$,

Digital image denoising scheme of this algorithm

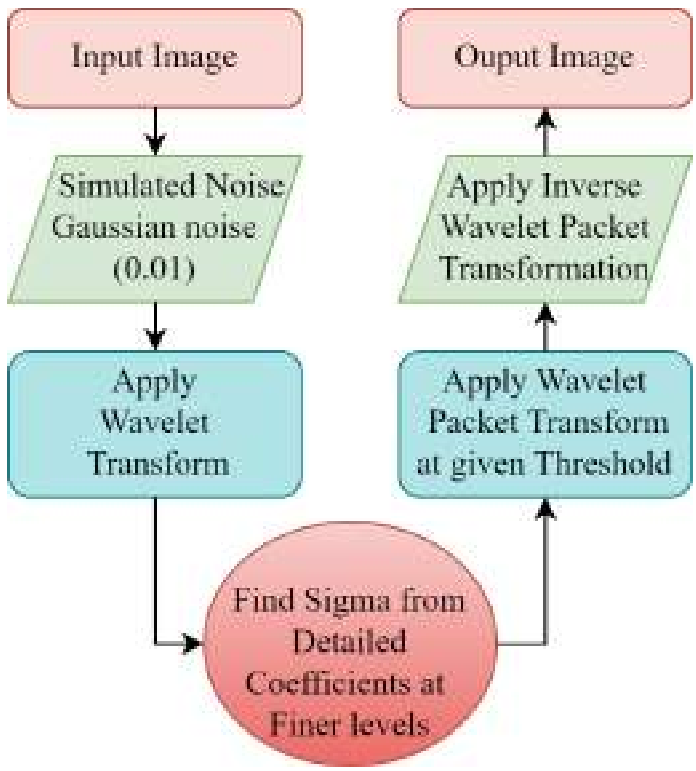

Fig.4.2: Diagram of a Proposed Algorithm

According to several research articles we have selected the Input Image as Lena 256×256 as our test image[55],[56],[57],[58],[59].

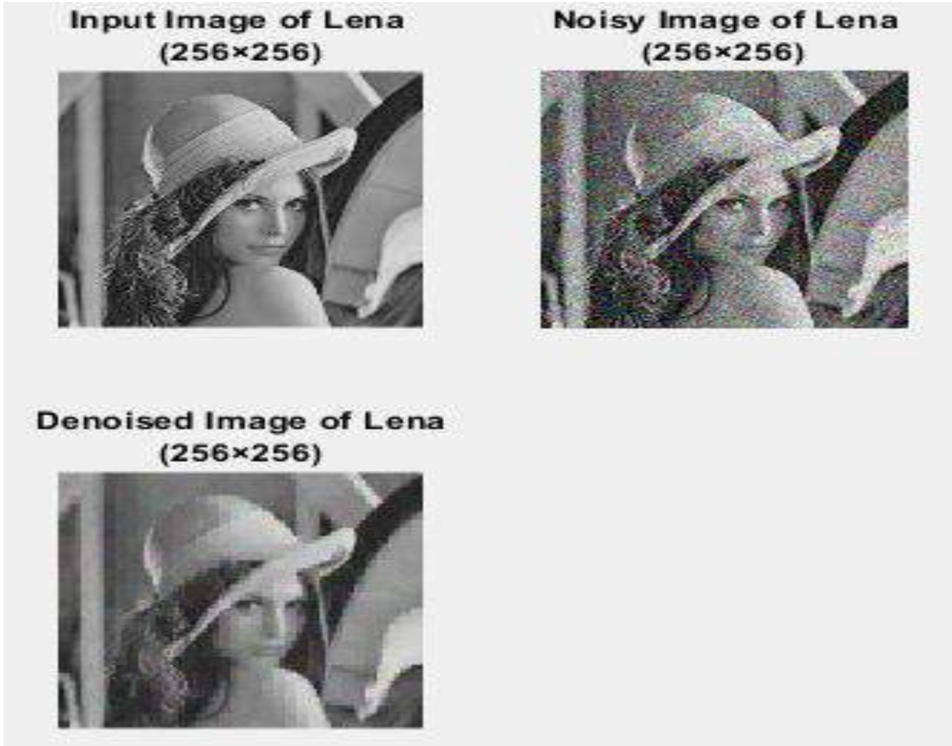

Fig.4.3: Input Image, Noisy Image, and Denoised Image of Lena

4.4 . Image Denoisng Analysis

Wavelet Packet Transform (haar)			
Digital image (256×256)	Simulated Noise Added	Noise variance obtained from (HL,LH,HH)	Threshold Value
Lena Grayscale	Gaussian Noise (0.01)	27.4277	44.6047

Table no.4.1: Wavelet Packet Transform (haar)

MSE,SNR and PSNR values			
Original **v/s** Denoised Image	MSE	SNR	PSNR
	51.6607	19.0837	29.6417

Table no.4.2: MSE,SNR and PSNR through WPT (haar) at $\lambda = 44.6047$

Wavelet Packet Transform (db5)			
Digital image 256×256	Simulated Noise Added	Noise variance obtained from (HL,LH,HH)	Threshold Value
Lena Grayscale	Gaussian Noise (0.01)	26.2614	42.7079

Table no.4.3: Wavelet Packet Transform (db5)

MSE,SNR and PSNR values			
Original **v/s** Denoised Image	MSE	SNR	PSNR
	50.6151	20.1456	30.7050

Table no. 4.4: MSE, SNR and PSNR through WPT (db2) at $\lambda = 42.7079$

Wavelet Packet Transform (sym7)			
Digital image 256×256	Simulated Noise added	Noise variance obtained from (HL,LH,HH)	Threshold Value
Lena Grayscale	Gaussian Noise (0.01)	26.1230	43.4729

Table no.4.5**:** Wavelet Packet Transform (sym7)

MSE,SNR and PSNR values			
Original **v/s** Denoised Image	MSE	SNR	PSNR
	49.7908	20.2572	30.7763

Table no.4.6: MSE, SNR and PSNR through WPT (sym7) at $\lambda = 43.4729$

Wavelet Packet Transform (coif2)			
Digital image 256×256	Simulated Noise added	Noise variance obtained from (HL,LH,HH)	Threshold Value
Lena Grayscale	Gaussian Noise (0.01)	26.2916	42.4829

Table no.4.7: Wavelet Packet Transform (coif2)

MSE,SNR and PSNR values			
Original **v/s** Denoised Image	MSE	SNR	PSNR
	49.3632	20.2910	30.9517

Table no.4.8: MSE, SNR and PSNR through WPT (coif2) at $\lambda = 42.4829$

Wavelet Packet Transform (bior6.8)			
Digital image 256×256	Simulated Noise added	Noise variance obtained from (HL,LH,HH)	Threshold Value
Lena Grayscale	Gaussian Noise(0.01)	25.8217	41.9924

Table no.4.9: Wavelet Packet Transform (bior6.8)

MSE,SNR and PSNR values			
Original **v/s** Denoised Image	MSE	SNR	PSNR
	50.6065	20.1627	30.7895

Table no.4.10: MSE, SNR and PSNR through WPT (coif2) at $\lambda = 41.9924$

	MSE,SNR and PSNR values between original and denoised images at different wavelets			
S.no's.	Wavelets	MSE	SNR	PSNR
1).	haar	51.6607	19.0837	29.641
2).	db5	50.6151	20.1456	30.7050
3).	sym7	49.7908	**20.2572**	**30.7763**
4).	coif2	49.3632	**20.2910**	**30.9517**
5).	bior6.8	50.6065	**20.1627**	**30.7895**

Table no.4.11: MSE, SNR and PSNR values of Lena Original and Denoised Image

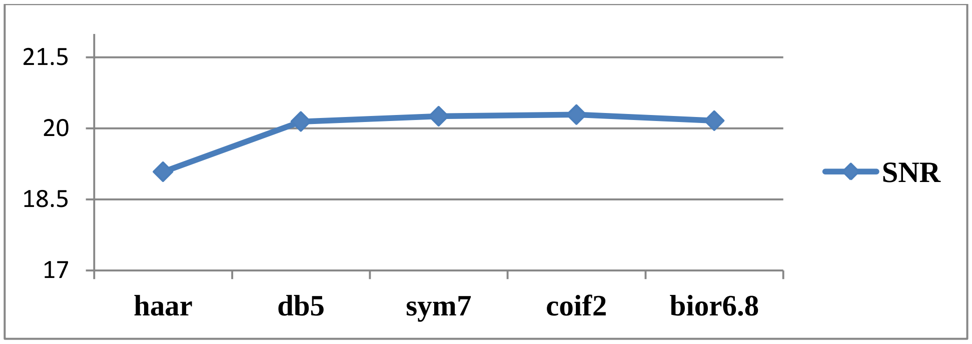

Fig.4.4: Line graph of SNR values of Lena Image

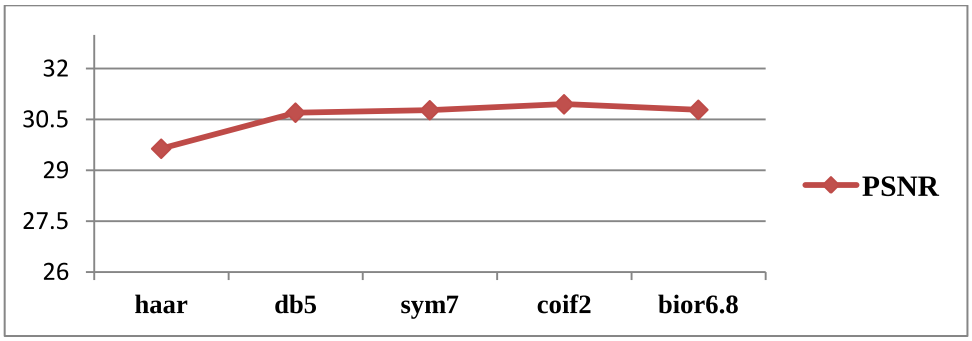

Fig.4.5: Line graph of PSNR values of Lena Image

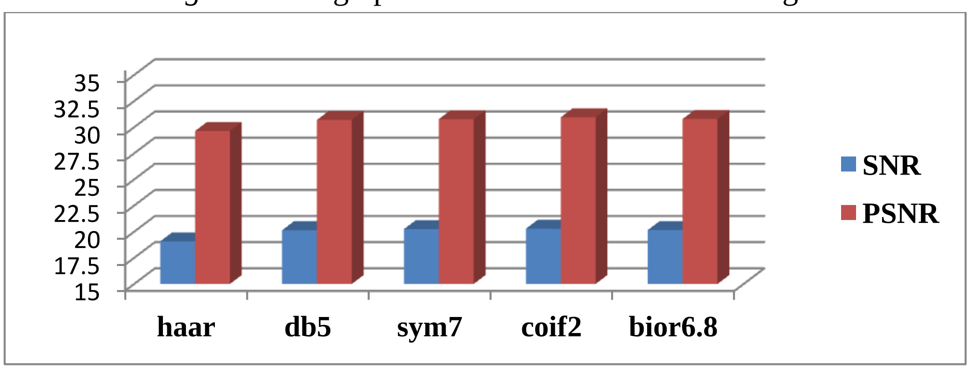

Fig.4.6: Column graph of both SNR & PSNR values of Lena Image

4.5 Summary

In this piece of work, we have selected a digital image of Lena of size 256×256 as our test image. We introduce a new thresholding function-based universal thresholding for the denoising of the above-mentioned image. The selected images are contaminated by gaussian white noise (GWN @ 0.01). First, we decompose the signal by wavelet transform at level-3 to get the value of sigma (σ) from detailed coefficients for the new thresholding function. Applied the wavelet packet transformation for the same image decomposition at the obtained threshold value for the image noise removal purposes. Then applied the IWPT at the same decomposition level to the above-decomposed image by WPT to obtain the reconstructed image or noise-free image. Finally, on the tabulated data analyses of SNR value between the input and reconstructed images. We analyze coif2 performs better in terms of image denoising at the first position and sym7 at the second position. Similarly, the wavelet functions bior6.8, db5, and haar performed better in terms of digital image denoising at the 3^{rd}, 4^{th}, and 5^{th}, positions. On other side the wavelet coif2, bior6.8, sym7, db5 and haar performs better in terms of PSNR values at 1^{st} ,2^{nd}, 3^{rd},4^{th},and 5^{th}, positions simultaneously.

Chapter 5

IMPORTANCE OF NORM IN A DIGITAL IMAGE DENOISING

5.1 Introduction

We have already mentioned in the preceding chapters of this thesis that digital images are a big source of information in different areas. In the modern world, digital images are captured and transferred at a very high speed for various purposes. However, various technical faults and human negligence, digital images may be corrupted by the wrong angle of the camera while capturing it. Sometimes these may catch noise (unwanted signal) by capturing them under poor lighting or at night[71],[72]. It may be corrupted due to the installed camera sensors like, "charge-coupled device (CCD), complementary metal oxide semiconductor (CMOS), electron-multiplying charge-coupled device (EMCCD), back-illuminated CMOS," etc.[73],[74]. Here we discuss some basic reasons for the corruption of digital images[72]. But there are other several reasons behind the corruption in digital images[72],[75]. Therefore, noise removal from these noisy images is one of the challenging task for the researcher and experts. The various research articles, clearly show that digital image denoising is a major trend in modern research[75]. Different approaches and techniques have been adopted to make them noise free[72],[76].

5.2 Digital Noise Addition

Usually, we insert different types of simulated noises in the input images for image denoising purposes[72]. As we have mentioned above, that digital image catches noise due to certain reasons. These noises may affect the part or the whole image and is a big challenge for researchers to remove them. In this chapter, we have selected Gaussian White Noise (GWT=0.003) as our simulated noise[72],[76],[77]. For denoising of the digital image, we insert selected simulated noise in the selected digital image at step3, according to our proposed algorithm. Mathematically can be defined as[77],[6],[72];

$$g(x, y) = f(x, y) + h(x, y) \qquad .... (5.1)$$

$$g(x, y) = f(x, y) + 0.003 \qquad .... (5.2)$$

Here $h(x, y)$ gaussian noise $f(x, y)$ is the original noise and $g(x, y)$ is a noisy image.

In the same way, we can multiply the same noise to the given input signal, i.e.

$$g(x, y) = f(x, y) \times h(x, y) \qquad ....$$

(5.3)

$$g(x, y) = f(x, y) \times 0.003 \qquad .... (5.4)$$

5.3 Norm

Mathematically, a Norm is a function that gives an absolute (non-negative) value to each vector (of any dimension) from a real or complex space. According to the literature, various types of norms are available for different purposes. It is sometimes used to calculate the distance between the coordinates or calculate the whole variation between two images[57],[43]. In this chapter of our thesis, we have used 1- Norm and 2-Norm as a tool to check out the variation between the noisy and the synthesized digital images through the proposed algorithm. If we have a 256×256 image it means there are 65536 dimensions or pixels in this image. So here norm may calculate the square root of the squares or pixels. The 1-Norm and 2-Norm are defined below as;

5.3.1 1-Norm

$$\|y\|_1 = |y_1| + |y_2| + |y_3| + |y_4| + ... + |y_n| \quad n \in Z \qquad .... (5.10)$$

Here $\|\ \|$ (double mod) represents the notation of the Norm, $|\ |$ represents modulus which provides only positive values to each vector and $\|y\|_1$ is called 1-Norm. It is sometimes denoted by L^1 norm or ℓ^1 norm which may calculate the total variation between the two images or measure the added average variation.

5.3.2 2-Norm

This Norm is identified by different names like Euclidean norm, L^2 norm, ℓ^2 norm or square norm etc. Sometimes it is a "square root of the inner product of a vector" with itself. This norm is mathematically defined as;

$$\|y\|_2 = \sqrt{|y_1|^2 + |y_2|^2 + |y_3|^2 + |y_4|^2 + ... + |y_n|^2} \qquad n \in Z \qquad .... (5.11)$$

5.3.3 Properties of Norm

There are various properties of the Norm, but some important are mentioned below as;

- $\|y\| \geq 0$
- $\|y\| = 0 \ \ iff \ \ y = 0$, here 0 is a $\vec{0}$ (vector).
- $\|ty\| = |t|\|y\|$, t is a scalar quantity and $|t|$ is the modulus of t, which gives a strictly positive value.
- $\|z + y\| \leq \|z\| + \|y\|$ (Triangular Inequality)

5.4 Proposed Algorithm

The digital image denoising is done under certain steps;

1) Load a digital image in MATLAB 2020a software.

2) Convert the input image into a grayscale image

3) Add a simulated noise (gaussian noise@ 0.003)

4) Apply the wavelet transform to decompose the input image at up to level-5, also applying the threshold (gbl) obtained from detailed coefficients at each level.

5) Finally, applying the Inverse Wavelet Transform to reconstruct the denoised digital image or to obtain the synthesized image at each level.

5.5 Denoising Performance Parameters

Once we select our test image and complete our image denoising process, we may obtain synthesizes image (noise-free image). Then we need to check the quality performance between our input image and output image (synthesized image). There are several parameters applied for the same purpose like MSE, PSNR, SNR, etc[76]. We have also introduced another parameter to check denoising performance between input and output digital images.

Here;

- $$SNR = 10\log\left[\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f\ (x,y)]^2 \Big/ \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[f(x,y) - f(x,y)\right]^2\right] \quad \text{................ (5.5)}$$

- $$MSE = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[f(x,y) - f(x,y)\right]^2 \Big/ M \times N \quad \text{................ (5.6)}$$

- $$PSNR = 10\log10(Max_i^{\,2}/MSE) = 20\log10(Max_i) - 10\log(MSE) \quad \text{................ (5.7)}$$

- 1-Norm= Max (Sum (abs (Original Image–Synthesized Image))) .... (5.8)

- 2-Norm =Max (sqrt (sum (abs (Original –Synthesized))^2) .... (5.9)

Here $f(x,y)$ is an input image, $f(x,y)$ is a synthesized image, Max_i is the maximum value in the signal i.e. if we have a digital image of size 256×256, then $Max_{i=}$ 256 and $(Max_i)^2 = (256)^{2,}$ Also Max in equation (5.8), (5.9) means the fluctuation of pixel values between the input and output images.

The digital image denoising scheme of this algorithm is shown below;

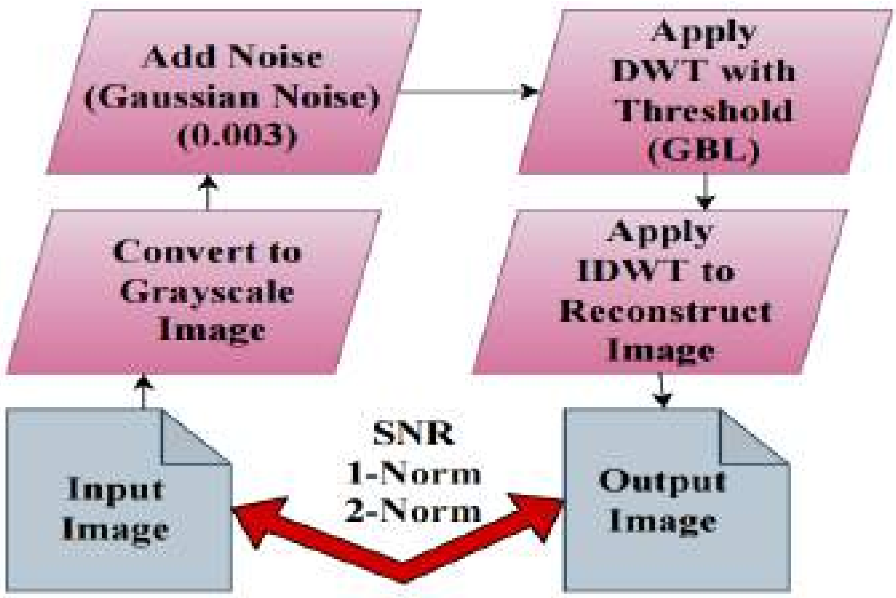

Fig.5.1: Diagram of the Proposed Algorithm

5.6 Table Analyses of the Results

Wavelet Transform (db2) with thresholding (gbl) obtained from detailed coefficients				
Input Image **(Barbara Image)** of size **512**$\times$**512**				
Decomposition Levels	L_1	L_2	L_3	L_4
SNR	18.5133	17.5905	16.5987	16.0952
1-Norm (N_1)	6944	7862	7841	8993
2-Norm (N_2)	208.5138	261.0268	260.1826	269.8222

Table no.5.1: SNR, 1-Norm, and 2-Norm values using wavelet (db2) at different decomposition levels of Barbara Image

Wavelet Transform (sym5) with thresholding (gbl) obtained from detailed coefficients				
Input Image **(Barbara Image)** of size 512×512				
Decomposition Levels	L_1	L_2	L_3	L_4
SNR	18.6542	17.6192	16.9241	16.4239
1-Norm (N_1)	5910	7249	7820	7938
2-Norm (N_2)	219.0183	245.4384	262.0868	265.1424

Table no.5.2: SNR, 1-Norm, and 2-Norm values using wavelet (sym5) at different decomposition levels of Barbara Image

Wavelet Transform (coif4) with thresholding (gbl) obtained from detailed coefficients				
Input Image **(Barbara Image)** of size 512×512				
Decomposition Levels	L_1	L_2	L_3	L_4
SNR	18.6721	17.6449	16.9814	16.4589
1-Norm (N_1)	5997	7360	7747	8197
2-Norm (N_2)	223.3449	231.3979	257.6645	266.4714

Table no.5.3: SNR, 1-Norm, and 2-Norm values using wavelet (coif4) at different decomposition levels of Barbara Image

Wavelet Transform (bior2.4) with thresholding(gbl) obtained from detailed coefficients				
Input Image **(Barbara Image)** of size 512×512				
Decomposition Levels	L_1	L_2	L_3	L_4
SNR	18.5057	17.6601	17.1460	16.7995
1-Norm (N_1)	7681	7573	7678	8235
2-Norm (N_2)	210.7724	252.7251	269.2341	275.5086

Table no.5.4: SNR, 1-Norm, and 2-Norm values using wavelet (bior2.4) at different decomposition levels of Barbara Image

SNR values between the original and denoised images (**Barbara Image)** at various decomposition levels				
Four levels of Decomposition	Wavelet (db2) SNR	Wavelet (sym5) SNR	Wavelet(coif4) SNR	Wavelet(bior2.4) SNR
L_1	18.5133	18.6542	18.6721	18.5057
L_2	17.5905	17.6192	17.6749	17.6601
L_3	16.5987	16.9241	16.9814	17.1460
L_4	16.0952	16.4239	16.4589	16.7995

Table no.5.5: SNR values using different wavelets for Barbara Image at different decomposition levels

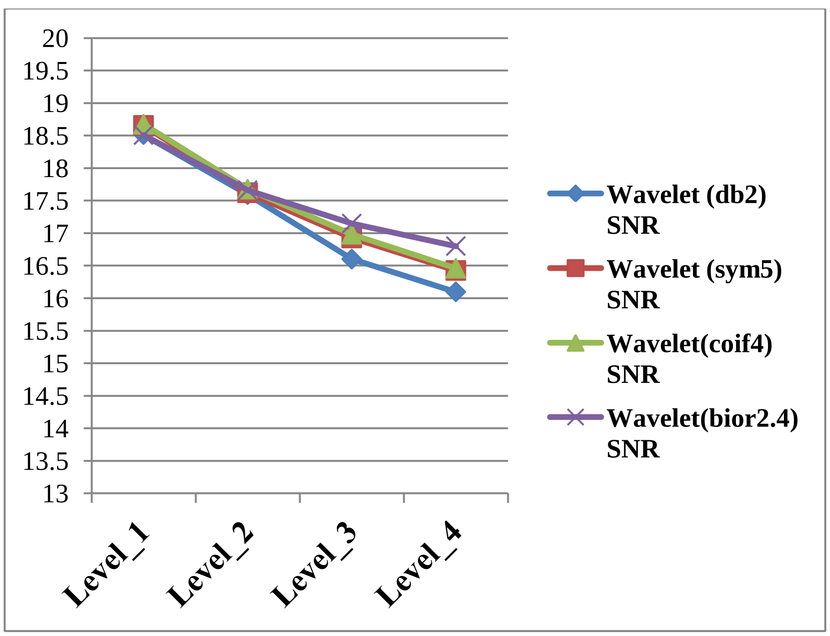

Fig.5.2: SNR values of four wavelets at different decomposition levels of Barbara Image

1-Norm (N_1) and 2- Norm (N_2) between the original and denoised images (**Barbara Image)** at various decomposition levels								
Four levels of Decomposition	Wavelet (db2)		Wavelet (sym5)		Wavelet (coif4)		Wavelet (bior2.4)	
	N_1	N_2	N_1	N_2	N_1	N_2	N_1	N_2
L_1	6944	208.5138	5910	219.0183	5997	223.3449	7681	210.7724
L_2	7862	261.0268	7249	245.4384	7360	231.3979	7573	252.7251
L_3	7841	260.1826	7820	263.0868	7747	257.6645	7678	269.2341
L_4	8993	269.8222	7938	265.8101	8197	266.4714	8235	275.5086

Table no.5.6:1-Norm and 2- Norm between the original and denoised images of Barbara Image

1-Norm (N_1)				
Decomposition Levels	(db2)	(sym5)	(coif4)	(bior2.4)
L_1	6944	5910	5997	7681
L_2	7862	7249	7360	7573
L_3	7841	7820	7747	7678
L_4	8993	7938	8197	8235

Table no.5.7: 1-Norm values for different wavelets at different decomposition levels of Barbara Image

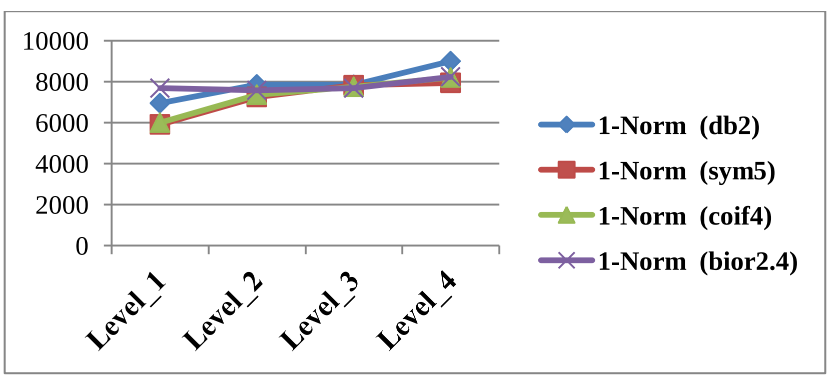

Fig.5.3: 1-Norm of four deferent wavelets at different decomposition levels of Barbara Image

2- Norm (N_2)				
Decomposition Levels	(db2)	(sym5)	(coif4)	Bior2.4
L_1	208.5138	219.0183	223.3449	210.7724
L_2	261.0268	245.4384	231.3979	252.7251
L_3	260.1826	263.0868	257.6645	269.2341
L_4	269.8222	265.8101	266.4714	275.5086

Table no.5.8: 2-Norm for different wavelets at different levels of Barbara Image

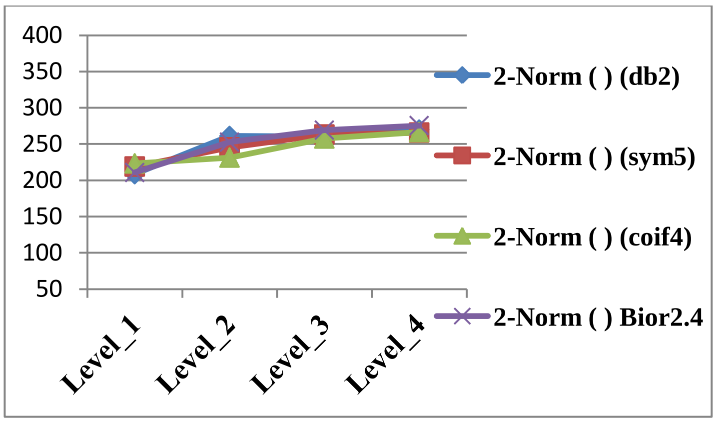

Fig.5.4: 2-Norm of four deferent wavelets at different decomposition levels of Barbara Image

Wavelet Transform (db2) with thresholding (gbl) obtained from detailed coefficients				
Input Image **(House Image)** of size **256**$\times$**256**				
Decomposition Levels	L_1	L_2	L_3	L_4
SNR	24.5304	23.6955	22.2280	21.4642
1-Norm (N_1)	3065	5011	6237	6969
2-Norm (N_2)	180.0805	203.7548	205.0293	208.3795

Table no.5.9: SNR, 1-Norm, and 2-Norm values using wavelet (db2) at different decomposition levels of House Image

Wavelet Transform (sym5) with thresholding(gbl) obtained from detailed coefficients				
Input Image **(House Image)** of size $\mathbf{256 \times 256}$				
Decomposition Levels	L_1	L_2	L_3	L_4
SNR	25.3318	25.0354	23.2804	22.2595
1-Norm (N_1)	4980	5176	5870	6370
2-Norm (N_2)	176.1448	202.3734	203.6050	206.9807

Table no.5.10: SNR, 1-Norm, and 2-Norm values using wavelet (sym5) at different decomposition levels of House Image

Wavelet Transform (coif4) with thresholding (gbl) obtained from detailed coefficients				
Input Image **(House Image)** of size $\mathbf{256 \times 256}$				
Decomposition Levels	L_1	L_2	L_3	L_4
SNR	25.3434	24.7842	23.0369	22.0753
1-Norm (N_1)	4583	5898	5996	6010
2-Norm (N_2)	199.2561	202.3018	202.9000	206.7801

Table no.5.11: SNR, 1-Norm, and 2-Norm values using wavelet (coif4) at different decomposition levels of House Image

Wavelet Transform (bior2.4) with thresholding(gbl) obtained from detailed coefficients				
Input Images **(House Image)** of size $\mathbf{256 \times 256}$				
Decomposition Levels	L_1	L_2	L_3	L_4
SNR	24.8597	24.6245	23.4488	22.8087
1-Norm (N_1)	2997	3600	3697	3710
2-Norm (N_2)	174.1608	195.4047	196.5310	198.5331

Table no.5.12: SNR, 1-Norm, and 2-Norm values using wavelet (bior2.4) at different decomposition levels of House Image

SNR values between the original and denoised images (**House Image)** at various decomposition levels				
Four levels of Decomposition	Wavelet (db2) SNR	Wavelet (sym5) SNR	Wavelet(coif4) SNR	Wavelet(bior2.4) SNR
L_1	24.5304	25.3318	25.3434	24.8597
L_2	23.6955	25.0354	25.7042	24.6245
L_3	22.2280	23.2804	23.5369	23.4488
L_4	21.4642	22.2595	22.7753	22.8087

Table no.5.13: SNR values using different wavelets for House Image at different decomposition levels

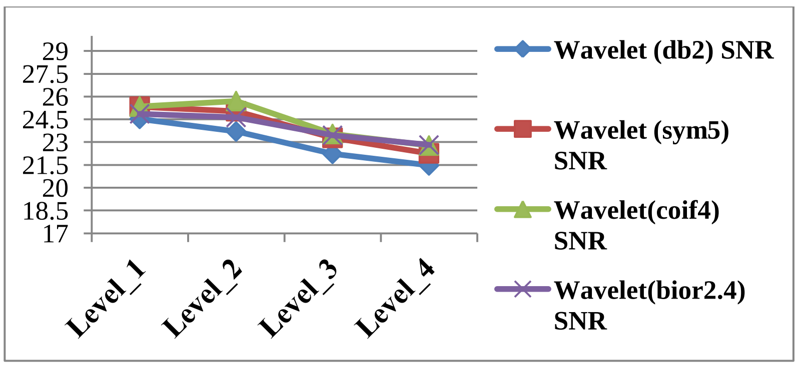

Fig.5.5: SNR values of four wavelets at different decomposition levels of House Image

1-Norm (N_1) and 2- Norm (N_2) between the original and denoised images (**House Image)** at various decomposition levels								
Four levels of Decomposition	Wavelet (db2)		Wavelet (sym5)		Wavelet (coif4)		Wavelet (bior2.4)	
	N_1	N_2	N_1	N_2	N_1	N_2	N_1	N_2
L_1	3065	180.0805	4980	180.0805	4583	199.2561	2997	174.1608
L_2	5011	203.7548	5176	203.7548	5898	202.3018	3600	195.4047
L_3	6237	205.0293	5870	205.0293	5996	202.9000	3697	196.5310
L_4	6969	208.3795	6370	208.3795	6010	206.7801	3710	198.5331

Table no.5.14:1-Norm and 2- Norm between the original and denoised images of House Image

1-Norm (N_1)				
Wavelet Decomposition Levels	(db2)	(sym5)	(coif4)	(bior2.4)
L_1	3065	4980	4583	2997
L_2	5011	5176	5898	3600
L_3	6237	5870	5996	3697
L_4	6969	6370	6010	3710

Table no.5.15: 1-Norm values for different wavelets at different decomposition levels of House Image

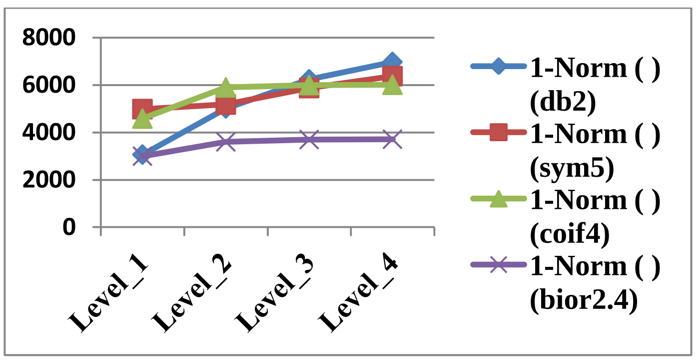

Fig.5.6: 1-Norm of four deferent wavelets at different decomposition levels of House Image

2- Norm (N_2)				
Wavelet Decomposition Levels	(db2)	(sym5)	(coif4)	Bior2.4
L_1	180.0805	176.1448	199.2561	174.1608
L_2	203.7548	202.3734	202.3018	195.4047
L_3	205.0293	203.650	202.9000	196.5310
L_4	208.3795	203.9807	206.7801	198.5331

Table no.5.16: 2-Norm values for different wavelets at different decomposition levels of House Image

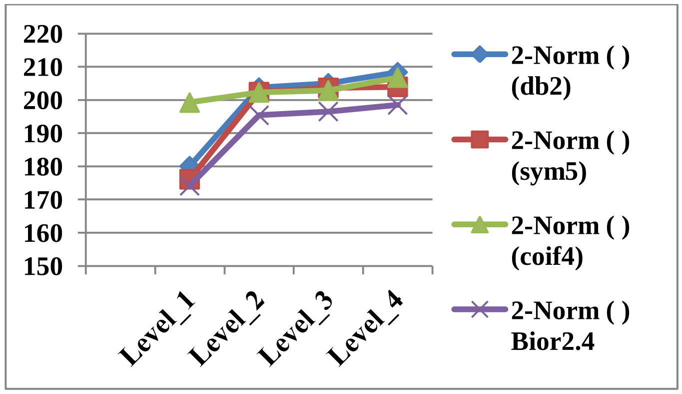

Fig.5.7: 2-Norm of four deferent wavelets at different decomposition levels of Barbara Image

Wavelet Transform (db2) with thresholding (gbl) obtained from detailed coefficients				
Input Image **(Author Image)** of size **256** $\times$ **256**				
Decomposition Levels	L_1	L_2	L_3	L_4
SNR	22.2954	20.4288	19.2570	18.7391
1-Norm (N_1)	2516	2632	2884	3030
2-Norm (N_2)	139.7891	148.5879	163.3524	165.5600

Table no.5.17: SNR, 1-Norm, and 2-Norm values using wavelet (db2) at different decomposition levels of Author Image

Wavelet Transform (sym5) with thresholding (gbl) obtained from detailed coefficients				
Input Image **(Author Image)** of size **256** $\times$ **256**				
Decomposition Levels	L_1	L_2	L_3	L_4
SNR	22.7575	20.8172	19.6321	19.0792
1-Norm (N_1)	1792	2175	2744	3152
2-Norm (N_2)	140.2712	151.4265	163.3952	168.2260

Table no.5.18: SNR, 1-Norm, and 2-Norm values using wavelet (sym5) at different decomposition levels of Author Image

Wavelet Transform (coif4) with thresholding (gbl) obtained from detailed coefficients				
Input Image **(Author Image)** of size **256**$\times$**256**				
Decomposition Levels	Level_1	Level_2	Level_3	Level_4
SNR	22.8036	20.8845	19.6518	19.0600
1-Norm (N_1)	1882	2502	2702	28.26
2-Norm (N_2)	139.8285	147.3839	161.1087	167.2393

Table no.5.19: SNR, 1-Norm, and 2-Norm values using wavelet (coif4) at different decomposition levels of Author Image

Wavelet Transform (bior2.4)with thresholding(gbl) obtained from detailed coefficients				
Input Image **(Author Image)** of size **256**$\times$**256**				
Decomposition Levels	L_1	L_2	L_3	L_4
SNR	22.5169	21.0364	20.1910	19.8172
1-Norm (N_1)	2376	2826	2989	3027
2-Norm (N_2)	142.5693	146.4138	154.3470	155.3158

Table no.5.20: SNR, 1-Norm, and 2-Norm values using wavelet (bior2.4) at different decomposition levels of Author Image

SNR values between the original and denoised image (**Author Image)** at various decomposition levels				
Four levels of Decomposition	Wavelet (db2) SNR	Wavelet(sym5) SNR	Wavelet(coif4) SNR	Wavelet(bior2.4) SNR
L_1	22.2954	22.7575	22.8036	22.5169
L_2	20.4288	20.8172	20.8845	21.0364
L_3	19.2570	19.6321	19.6518	20.1910
L_4	18.7391	19.0792	19.0900	19.8172

Table no.5.21: SNR values using different wavelets for Author Image at different decomposition levels

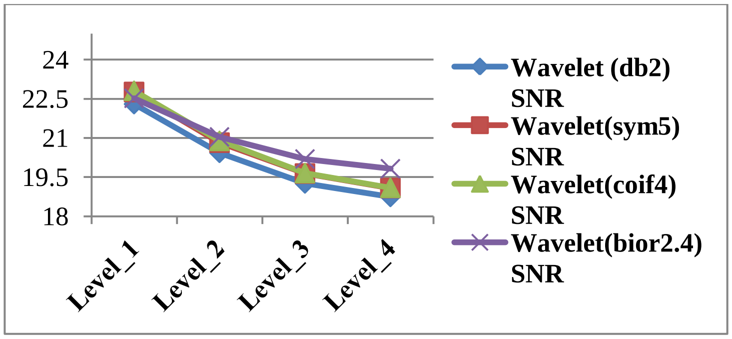

Fig.5.8: SNR values of four wavelets at different decomposition levels of Author Image

1-Norm (N_1) and 2- Norm (N_2) between the original and denoised image (**Author Image**) at various decomposition levels								
Four levels of Decomposition	Wavelet (db2)		Wavelet (sym5)		Wavelet (coif4)		Wavelet (bior2.4)	
	N_1	N_2	N_1	N_2	N_1	N_2	N_1	N_2
L_1	2516	139.7891	1792	140.2712	1882	139.8285	2376	142.5693
L_2	2632	148.5879	2175	151.4265	2502	147.3839	2826	146.4138
L_3	2884	163.3524	2744	163.3952	2702	161.1087	2989	154.3470
L_4	3030	164.5600	3152	168.2260	2826	167.2393	3027	155.3158

Table no.5.22:1-Norm and 2- Norm between the original and denoised images of Authors Image

1-Norm (N_1)				
Wavelet Decomposition Levels	(db2)	(sym5)	(coif4)	(bior2.4)
L_1	2516	1792	1882	2376
L_2	2632	2175	2502	2826
L_3	2884	2744	2702	2989
L_4	3030	3152	2826	3027

Table no.5.23: 1-Norm values for different wavelets at different decomposition levels of Author Image

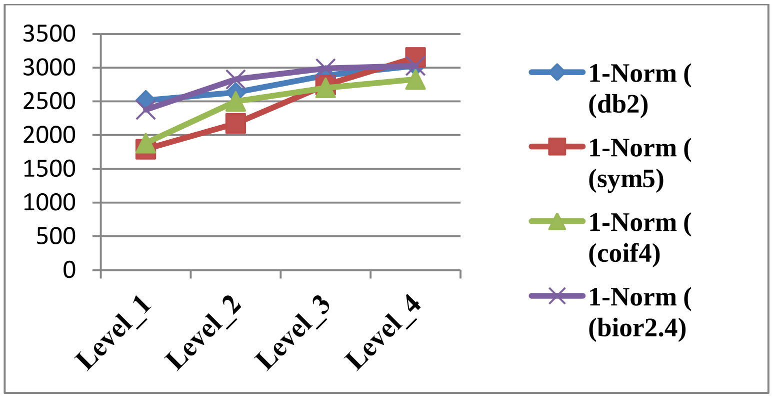

Fig.5.9: 1-Norm of four deferent wavelets at different decomposition levels of Author Image

2- Norm (N_2)				
Wavelet Decomposition Levels	(db2)	(sym5)	(coif4)	Bior2.4
L_1	139.7891	140.2712	139.8285	142.5693
L_2	148.5879	151.4265	147.3839	146.4138
L_3	163.3524	163.3952	161.1087	154.3470
L_4	164.5600	168.2260	167.2393	155.3158

Table no.5.24: 2-Norm values for different wavelets at different decomposition levels of Author Image

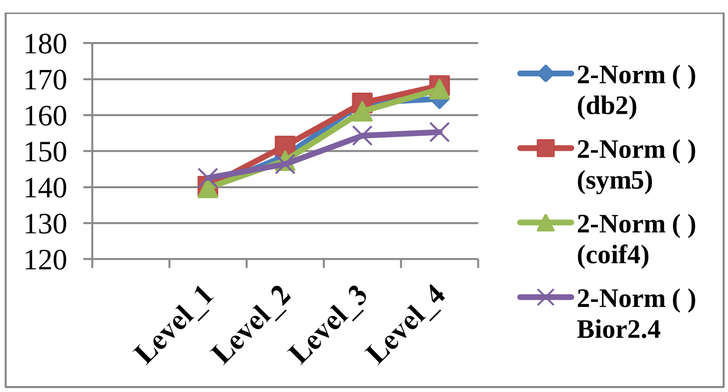

Fig.5.10: 2-Norm of four deferent wavelets at different decomposition levels of Author Image

5.7 Input Digital Images (or Test Images) and Denoised Digital Images

Here we have selected Barbara and House digital image as our test images. Because these images are used in different research papers for the same purpose and we have downloaded it online at the below links;

https://www.researchgate.net/figure/Barbara-256-256-Model-with-8-possible-angles-LeftLinearmodel14730_fig10_220411352","https://www.researchgate.net/profile/GuodongYe/publication/337655573/figure/fig4/AS:1086065335115779@1635949562478/Test-result-a-plain-image-of-house-b-carrier-image-of-landscape-c-secret-image.jpg"[78].

Here author selected his own digital image as third test image. Finally, the denoised images of these test images are given below as.

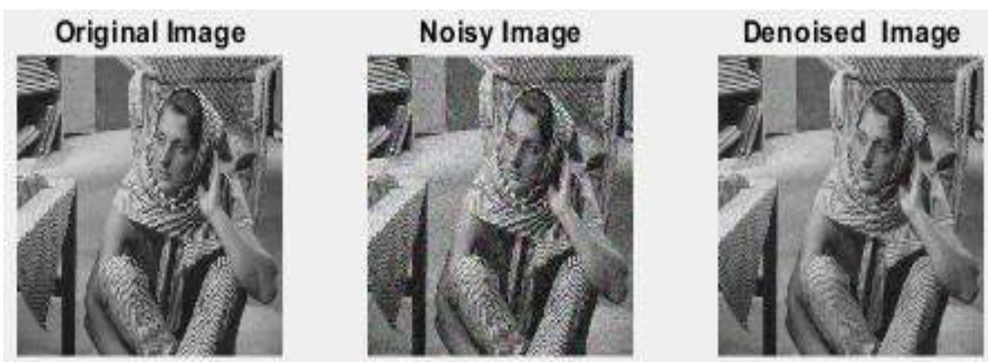

Fig.5.11: Barbara Digital Image and Synthesized Image

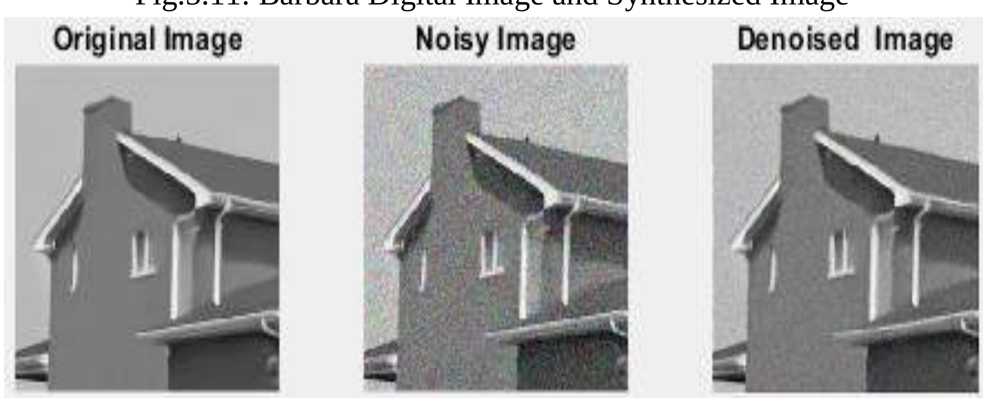

Fig.5.12: House Digital Image and Synthesized Image

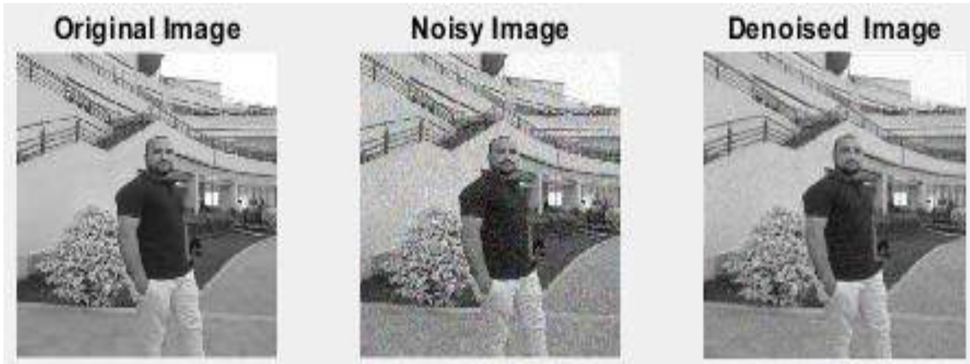

Fig.5.13: Author Digital Image and Synthesized Image

5.8 Summary

Like the previous chapters of this thesis, the whole work is done through MATLAB 2020a software under the steps of the proposed algorithm and performance parameters. Each digital image has its own properties and noises in it. So the true digital image denoising depends on the property of the selected digital image, the noise present in it, and the selected wavelet function. To denoise the digital images we have an open choice to select the decomposition level and obtain the threshold values at any decomposition level. With the help of wavelet transform we have decomposed each selected digital image up to four levels and used a global threshold at each level for image denoising. To check the digital image denoising by wavelets at each level, we select denoising performance parameter SNR.

From above table and figure i.e. Table no.5.5 and Fig.5.2 clearly shows each wavelet performs well at each level in terms of denoising of digital image of Barbara. And the table and figure i.e. Table no.5.13,and Fig.5.5 visualize each wavelet perform well in terms of denoising of test image of House. Also the Table no.5.21 and Fig.5.8 shows the better performance of each wavelet at each level for a test image of Author.

As mentioned above each wavelet performs better in terms of image denoising for each test image, but wavelet coif4 for some extent performs better than other wavelets.

On other side the variation parameter Norm shows the increasing variation between the original and denoised images. Both norms show better results in good way. The result of 1-norm for the Barbara, House and Author digital image is shown by the tables and figures i.e. Table no.5.6, Table no.5.15, Table no.5.23 and Fig.5.3, Fig.5.6, Fig.5.9. Which clearly show the variation between the input and output images increased from level to level. Also, from the above tables and figures i.e. Table no.5.8, Table no.5.16, Table no.5.24 and Fig.5.4, Fig.5.7, Fig.5.10, the 2-Norm to the input and output of the tests images of Barbara, House and Authors too shows variation among the input and output images increased from each wavelet decomposition level.

Conclusion

In this research, we demonstrate digital image denoising algorithms based on wavelet transformations and other several parameters to denoise grayscale images. Each parameter is defined in each proposed algorithms separately and the basic manual mathematical calculations were performed for different parameters before executing the algorithm.

Different grayscale digital images used in different research articles are used as test images for denoising motives, such as CT scan of the human chest, Cameraman, Lena, Barbara, and House images. Irrespective of these images, the author used his own grayscale digital image as a test image for the same purpose. The simulation for image denoising was performed in MATLAB using its wavelet analyzer and our own MATLAB programs. After that, a comparative study of the parameters used in these algorithms is performed using prominent performance parameters and other statistical terms.

The conclusion of this thesis is defined in a chapter-wise manner as;

Chapter 1: In this chapter a brief introduction of transformation is presented. A detailed review of relevant literature was conducted. Different techniques, approaches, methods, and algorithms have been noted. It has been noted that wavelet transformation plays a significant role not only in image denoising but is used for other purposes, such as image compression, image edge detection, image segmentation, image smoothing and sharpening, image spike and discontinuity detection, and image enhancement. In this chapter, the approach employed to accomplish the various stated objectives is covered. It largely covers the existence and applicability of wavelet theory, various wavelet functions, transformations, and the works involved in using wavelet functions to approximate an image. Wavelet packet transformation, inverse wavelet packet transformation, and multiresolution analysis (MRA) are discussed for signal analysis. Different programs were discussed for how to load images in MATLAB for denoising purposes. Further, the objectives of this research were developed in consideration of the available literature review. The future scope and organization of this thesis is summarized at the end of this chapter.

Chapter 2: The primary focus of this chapter is on the denoising digital image in detail. A CT-scan image of human chest is taken as test image for denoising and loaded it in MATLAB (2020a) software. The denoising and analyses of digital image is done according to proposed algorithm. Here, the input image was passed through four filters for denoising after being added to four synthetic noises. Then denoising of digital image is further done through wavelet transformation (DWT) and thresholding technique (gbl). The synthesized CT scan image was obtained by inverse wavelet transformation (IDWT). At end it was found that the denoising of digital image by wavelets i.e. sym4,coif2,db2 and bior1.5 with winner filter on poisson simulated noise performs better than other wavelets and filters.

Chapter 3: The algorithms proposed in this chapter mostly focus on the threshold function $\lambda(n) = \sigma\sqrt{n\log(M)} = T$, $n \varepsilon N$ for generalization. The digital image of cameraman is selected as test image and is decomposed up to level 3 by wavelet transformation. A denoising performance comparison is laid down for eleven wavelet function on the bases of signal noise ratio and peak signal noise ratio. This comparison is shown in terms of SNR, the average of three SNR values that were independently acquired using the soft thresholding technique and the given mathematical methods. It is shown that each wavelet performs better in terms of digital image denoising. Finally, the wavelet function bior3.3, in comparison with the other wavelet functions chosen for image denoising, performs better in terms of image denoising.

Chapter 4: In this chapter a digital image of Lena is taken as test images. The estimated threshold value is obtained from detailed coefficients at level three by general thresholding function for digital image denoising. Then the wavelet packet transformation with obtained threshold values is applied for digital image denoising at level-3.Finally, on the tabulated data analyses of SNR value between the input and reconstructed images. We analyze coif2 performs better in terms of image denoising at position first and sym7 at second position. Similarly, the wavelet functions bior6.8, db5, and haar performed better in terms of digital image denoising at the 3rd, 4th, and 5th, positions.

Chapter 5: In this chapter, the importance of the norm parameter in digital image denoising is discussed. The proposed approach also makes use of several parameters. Here we select three test images i.e. Barbara, House and Author digital image for denoising purposes. At the beginning the gives test image is decomposed by several wavelet functions at level-3. The threshold value is obtained from detailed coefficients of a image by thresholding function (gbl) for denoising of digital image. Additionally, the 1-Norm and 2-Norm expresses that all wavelet functions used gives better denoising performance and check out fluctuation increased among th e input and output images.

We find after each level of decomposition the denoising performance decreases gradually and fluctuation among then images increases rapidly. Finally, we observe each wavelet performs better in terms of image denoising, but coif4 for some extent performs better than other wavelet functions for all selected test images.

Future Scope of this Research Work

- In this thesis, we aim to study denoising of digital images, especially grayscale digital images.
- We extended our research to denoise other digital images, like RGB, Binary, Black and white etc.
- The latest literature clearly shows that the wavelet transformation is a best tool for not only digital image analyses but also for the analysis of other different types of signals
- It can be further extended in future works to denoise other types of signals, such as Biological Signals, Seismic Signals, and Musical Signals.
- We can also enhance the methodology of the wavelet transform by selecting good and perfect wavelet decomposition levels for a clear and broader analysis of different signals.
- One more key area for research that can enhance wavelet methodology is the selection of perfect threshold functions or threshold values.
- This work can be further extended to an important version of the wavelets called curvelet transformation, by which we can hold the information in a signal at the curve point, which is not retained by wavelet transformation

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Appendix

Paper Publications

1. Parvze Ahmad Dar[1], Afroz[2] and K.Khan[3], "**Comprehensive Study: Wavelet Transform for Digital Image Processing Like, Image Enhancement, Image compression and image De-Noising**", **Glob.sci.tech**, vol.12, no.2, pp.106-116, April-June, 2020.

2. Parvze Ahmad Dar[1], Afroz[2], **"Performance Comparison of Different Filter for Denoising the Digital Noisy Images through Wavelet Transformation", Design Engineering, Scopus Indexed Journal**, vol.8, pp.15789-15800, 21-Novemver, 2021, ISSN no.0011-9342.

3. Parvaze Ahmad Dar[1], Afroz[2], " **A new Thresholding Technique for Denoising of Digital Images by Using Wavelet Packet Transformation", International Journal of Mechanical Engineering, Scopus Indexed Journal**,vol.7,pp.4363-4372,2-February,2022,ISSN no.0974-5823.

4. Parvaze Ahmad Dar[1], Afroz[2], **" Estimation of Threshold for Denoising of Digital images Through wavelet Packet Transformation", Stochastic Modelling & Applications, UGC Care listed Journal and Scopus Indexed Journal**,vol.26,no.3, 8-June,2022,ISSN no. 0972-3641.

Paper Presentation:

1. Parvaze Ahmad Dar[1], " **National Conference on Emerging Trends and Issues in Information Technology & communication", "Performance comparison of different wavelet for ECG signal Denoising**", organised by school of science, MANUU, Hyderabad held on (17-18)th March,2018.

2. Parvaze Ahmad Dar[1], **"National Conference on Emerging Trends in Mathematics and its applications", "Wavelet Transform and Its Applications in Digital Image Processing"**, organised by Gitam School of Sciences, Department of Mathematics ,Bangalore on (20-21)th December,2019.

3. Parvaze Ahmad Dar[1], **"International Conference on, "Emerging Trends in Intelligent Information Technology/Applied Mathematics and Business Management", "Comprehensive Study: Wavelet Transform for Digital Image Processing Like, Image Enhancement, Image compression and Image De-Noising** "organised by the Quaide Milleth College for Men ,Chennai, on 7th to 8th January,2020.